Parametric Interpolation Scheme Based on Boundary Blending

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Scattered Data Problem

Given a triangular mesh of vertices, construct a smooth surface to

- interpolate the vertices.
- interpolate the first order derivatives/normals.

Nielson's Method

- Three vertices V_u, V_v, and V_w and their normals.
- Three parametric surfaces S_u, S_v, and S_w that satisfy

1. S_u, S_v, and S_w interpolate the locations of V_u, V_v, and V_w,
2. S_u, S_v, and S_w have the same boundaries,
3. S_u interpolates the normals at V_v and V_w,
4. S_v interpolates the normals at V_w and V_u,
5. S_w interpolates the normals at V_u and V_v.

- Blending functions:

$$\frac{vw}{uv+vw+wv},\quad \frac{wu}{uv+vw+wv},\quad \frac{uv}{uv+vw+wv}.$$

- Blended surface S:

i. S and S_u have the same tangent plane field along V_vV_w,
ii. S and S_v have the same tangent plane field along V_wV_u,
iii. S and S_u have the same tangent plane field along V_uV_v.

- Higher order continuity also works [1].

$$\frac{v^tw^t}{u^tv^t+v^tw^t+w^tv^t},\quad \frac{w^tu^t}{u^tv^t+v^tw^t+w^tv^t},\quad \frac{u^tv^t}{u^tv^t+v^tw^t+w^tv^t},$$

where t is the desired order of continuity.

Restrictions of Nielson's Method

Each of Nielson's blending functions does not have a limit at the three corners, that forces the three sub-surface to share the same boundaries.

- The three sub-surfaces used in Nielson's method must share the same boundaries.
- Determining the boundary curves before constructing sub-surfaces is necessary.

The Modified Method

- Relaxed conditions

Condition 1 is relaxed so that each surface is only required to interpolate the locations at two corners (the same corners as specified for normals in Conditions 3–5). Our new method no longer needs to meet Condition 2 above. Thus, the conditions on our subsurfaces are

3'. S_u interpolates the locations and normals of V_v and V_w,
4'. S_v interpolates the locations and normals of V_w and V_u,
5'. S_w interpolates the locations and normals of V_u and V_v,

- The new blending functions

$$\begin{aligned} f_{t,0} &= \beta\gamma\left(\frac{1}{\alpha+\beta}+\frac{1}{\alpha+\gamma}\right)\left(\frac{1}{\alpha+\beta+\gamma}\right),\\ f_{t,1} &= \gamma\alpha\left(\frac{1}{\beta+\gamma}+\frac{1}{\beta+\alpha}\right)\left(\frac{1}{\alpha+\beta+\gamma}\right),\\ f_{t,2} &= \alpha\beta\left(\frac{1}{\gamma+\alpha}+\frac{1}{\gamma+\beta}\right)\left(\frac{1}{\alpha+\beta+\gamma}\right), \end{aligned}$$

where $\alpha = u^{t+1}$, $\beta = v^{t+1}$, and $\gamma = w^{t+1}$, and t is a non-negative integer.

- Properties

The resulting blended surfaces will meet with C^t continuity.

Unlike Nielson's scheme, at each corner, the corner of one of the sub-surfaces is free to be placed anywhere.

Example

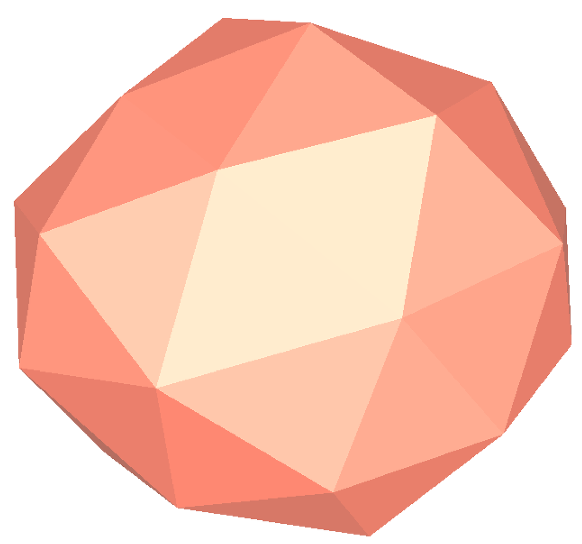

Figure 1: Test input mesh.

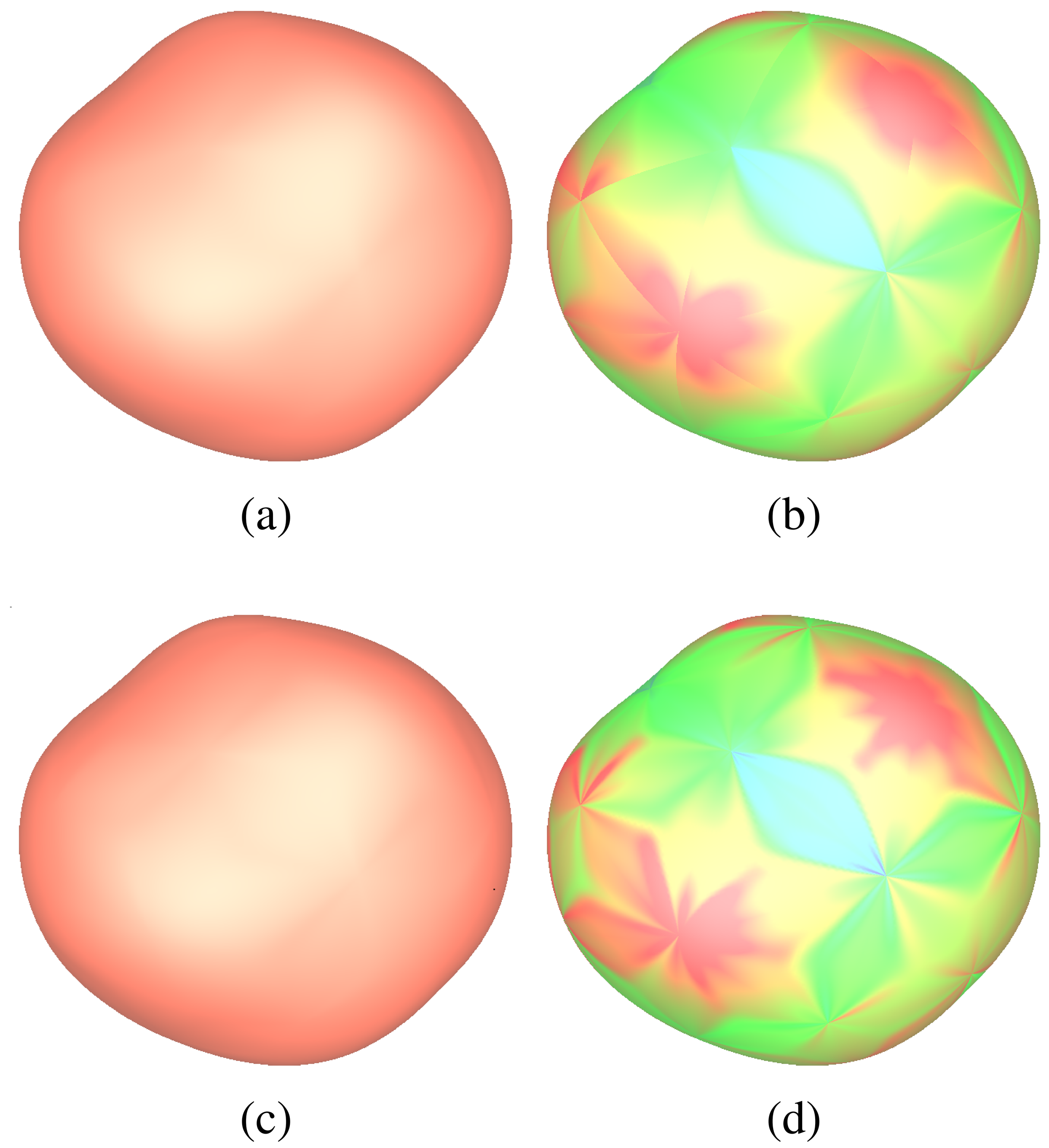

Figure 2: Blended surfaces and curvature plots.

Figure 2 shows two surfaces and Gaussian curvature plots constructed with our method. Figure 2 (a) and (b) show surfaces blended with cubic Bézier patches with $t = 1$; the patches meet with C^1 continuity. Figure 2 (c) and (d) show surfaces blended with cubic Bézier with $t = 2$; the patches meet with C^2 continuity. In the curvature plots, the red color stands for positive Gaussian curvature, the blue stands for negative, and the green stands for zero. It can be seen that the first surface is not C^2 continuous since the curvature is not continuous, but the curvature of the second surface is continuous across the boundaries.

References

[1] H. Hagen and H. Pottmann. Curvature continuous triangular interpolants. *Mathematical Methods in Computer Aided Geometric Design*, 1989.

[2] G.M. Nielson. A transfinite, visually continuous, triangular interpolant. *Geometric Modeling: Algorithms and New Trends*, 1987.