A Study of Two Implicit Data Interpolation Schemes

Stephen Mann
Computer Graphics Lab, David R. Cheriton School of Computer Science, University of Waterloo

Abstract

Implicit surfaces are one technique for surface modeling in computer graphics. The are several implicit surface schemes to interpolate point data. In this poster, I study two schemes, one by Turk and O'Brien, the other by Shen, and test how well they work.

For this work, I implemented 2D versions of their methods in Octave, and created a variety of data sets to test their method. Further, a few variations on both schemes were tested.

Implicit Functions

An implicit surface is defined as the zero set of a scalar function over space. The sign of the implicit function determines whether a point is inside or outside the implicit surface:

$$f(P)\begin{cases} =0 & P \text{ is on the surface.} \\ <0 & P \text{ is inside the surface.} \\ >0 & P \text{ is outside the surface.}\end{cases}$$

Interpolatory Implicit Functions

Given: a set of points $P = \{p_0, ..., p_n\}$

Find: F such that $F(p_i) = 0$

More generally, we may have a set of values $V = \{v_0, ..., v_n\}$ and want F such that $F(p_i) = v_i$.

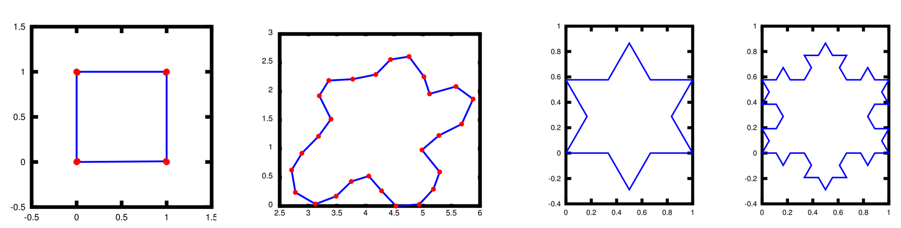
Criteria

In the tests, I was looking for several things including the following:

- Are there any shape artifacts in the curves?
- Is the curve a single, connected component?
- Does the curve follow the connectivity of the polyline defining the data?
- Are there extra sheets that do not interpolate the data?

Most of the evaluation of the curves is just a subjective, visual check of the shape. This is mostly sufficient in checking the last three questions above (although potentially there are disconnected components far from the data that were not detected).

Data



Turk-O'Brien

Given radial basis function $r_q(p)$, let

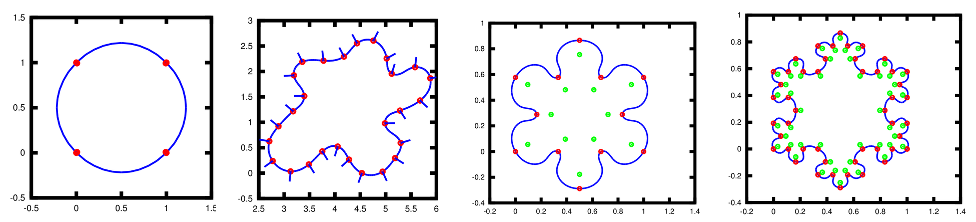
$$F(p) = \sum_i w_i r(p - p_i) + P(p),$$

where P is a linear polynomial. Construct system of equations $F(p_i) = v_i$ and solve for unknowns c_i. Turk-O'Brien use $r_q(p) = ||p - q||^3$.

Problem: if all $v_i = 0$ then system has trivial solution $w_i = 0$ for all i.

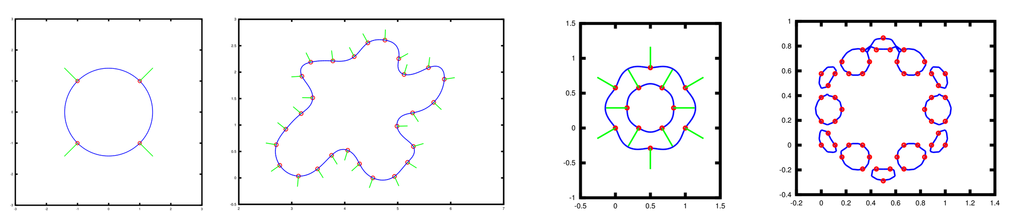
Turk-O'Brien choose additional locations off the zero set to interpolate. Initially they chose arbitrary locations/values, but later switched to *pseudo-normal constraints*, which for each p_i places a point at $p_i + 0.01\hat{n}_i$ with value 1.

The Turk-O'Brien method using pseudo-normals worked well: the curves constructed had reasonable shape and for reasonable data spacing, the curves had the same topology as the polyline. The only issue I encountered with their scheme (not seen in these examples) was that sometimes their method produced extra, unwanted sheets.



Turk-O'Brien using pseudo-normals.

I also tried using the true-normals of Shen with the Turk-O'Brien method with mixed results.



Turk-O'Brien using true-normals.

References

[1] K. Bibak, C. Liu, H. Vosoughpour, G. Yao, Z. AlMeraj, A. Pytel, W. Cowan, and S. Mann. Implicit surfaces seminar, spring 2012. Technical report, Cheriton School of Computer Science, University of Waterloo, 2013. in preparation.

[2] S. Mann. A study of two implicit data interpolation schemes. Technical report, Cheriton School of Computer Science, University of Waterloo, 2013. in preparation.

[3] C. Shen. *Building Interpolating and Approximating Implicit Surfaces Using Moving Least Squares*. PhD thesis, University of California, Berkeley, 2007. UCB/EECS-2007-14.

[4] G. Turk and J. F. O'Brien. Shape transformation using variational implicit functions. In *Proceedings of the 26th annual conference on Computer graphics and interactive techniques*, SIGGRAPH '99, pages 335–342, New York, NY, USA, 1999. ACM Press/Addison-Wesley Publishing Co.

[5] G. Turk and J. F. O'Brien. Modelling with implicit surfaces that interpolate. *ACM Trans. Graph.*, 21(4):855–873, 2002.

Shen

The idea of Shen is to use Moving Least Squares to compute an implicit function that interpolates a set of points. In particular, Shen used an error function

$$R(\mathbf{p}) = \sum_{i=1}^{N} w(||\mathbf{p} - \mathbf{p}_i||)[v_i - f(\mathbf{p_i})]^2, \qquad (1)$$

where $f(\mathbf{p_i})$ is a polynomial, where in the case of interpolation we want $f(\mathbf{p}_i) = v_i$, and where $w(r)$ is a weight function. Shen uses

$$w(r) = \frac{1}{r^2 + \epsilon^2},$$

where ϵ is a user defined parameter. When $\epsilon = 0$, the resulting implicit interpolates the data, and when $\epsilon \neq 0$, the implicit approximates the data. Regardless, with Shen's approach, you do not need to solve an $n \times n$ linear system of equations.

Although Shen derives formulas for f of degree one, in practise he uses f of degree zero.

True normals

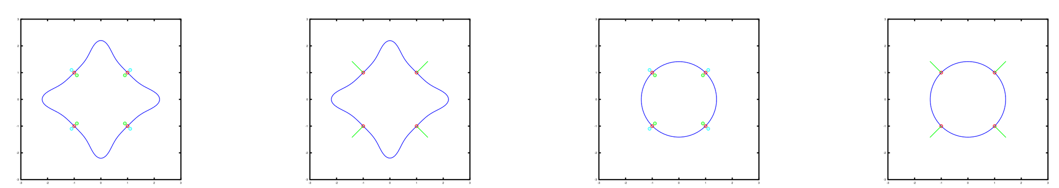
On seeing problems with his constant method using pseudo-normals, Shen introduced *true normals*. The idea is to express the normal as a linear implicit function, and to interpolate the coefficients of these linear functions.

Tests

I implemented two versions of Shen's scheme, one using constant f and one using linear f. I tested each version of f with both the pseudo-normals of Turk-O'Brien and with the true-normals of Shen, giving four variations on Shen's scheme.

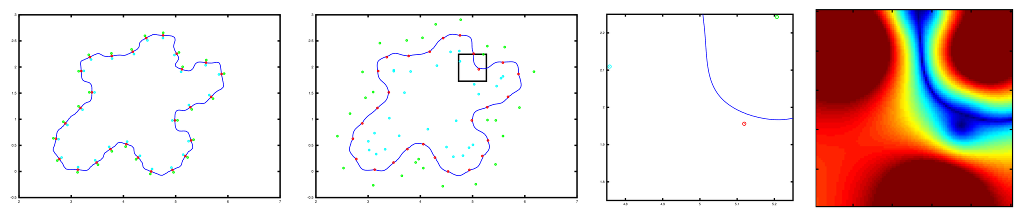
The results with all four variations of Shen's interpolation method were poor.[1] As seen in the examples, the resulting curves had flat spots, bumps, and numerical issues. (The numerical issues are due to Shen's linear method having to invert a 3×3 linear system that becomes unstable as the evaluation point approaches a data point.)

Circle data



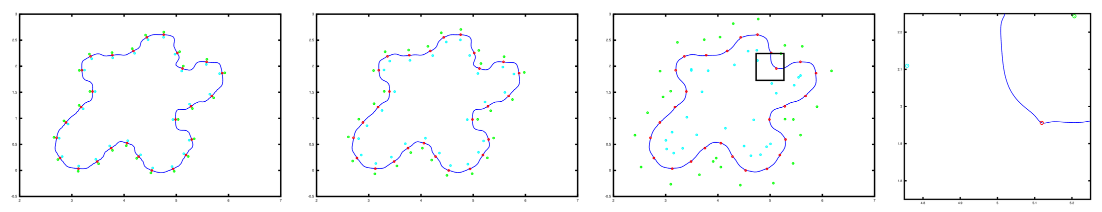
constant, pseudo-normals | constant, true-normals | linear, pseudo-normals | linear, true-normals

Shen data set



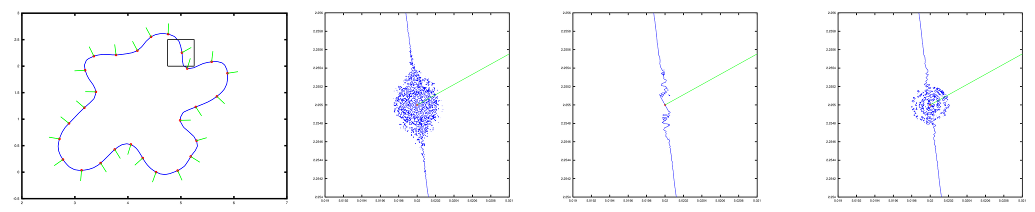
(a) (b) (c) (d)

Shen's constant method, pseudo-normals



(a) (b) (c) (d)

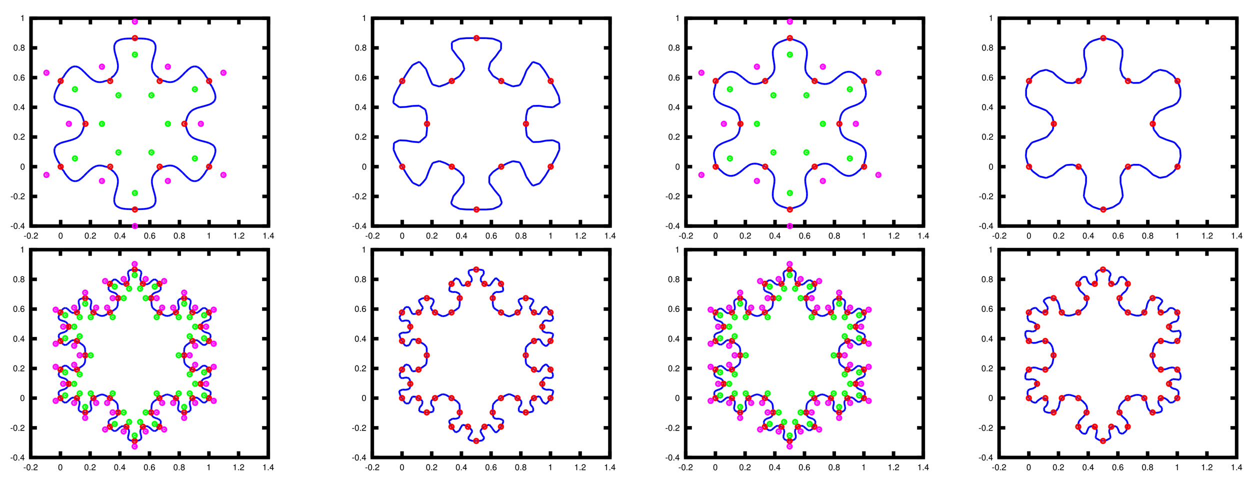
Shen's linear method, pseudo-normals



`inverse` `backslash` LU-decomp.

Shen's linear method, true-normals

Koch snowflake data



constant, pseudo-normals | constant, true-normals | linear, pseudo-normals | linear, true-normals

Conclusion

While the Turk-O'Brien scheme gives reasonable results for curves, Shen's scheme appears to have serious problems. Additional details of this study can be found in [2]. And although the tests run for this poster were for curves, both methods were proposed for constructing surfaces. Further tests would be needed to determine if either method constructs reasonable surfaces.

Finally, much work on interpolating implicit surfaces has been published since the two methods reviewed for this poster, as well there have been other evaluations of both schemes. This poster should be viewed as an additional series of tests of these two methods.

[1] The curves created by Shen's method to approximate the data had much better shape, although the parameters to control the approximation were difficult to set.