Approximate Continuity for Parametric, Triangular Bézier Surfaces

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Introduction

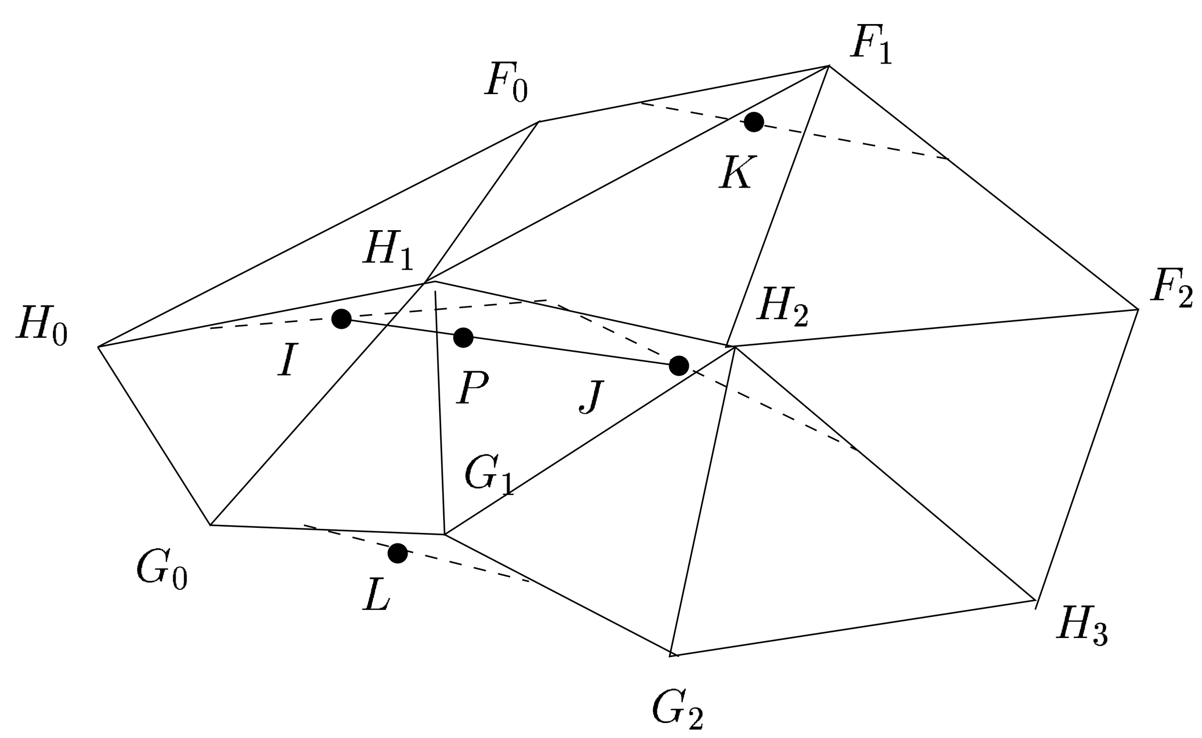
For two parametric triangular Bézier patches to meet each other with G^1 continuity, their control points have to fulfil certain constrains. These constraints will lead to high degree patches or bad shape properties. The idea of *approximate continuity* is not to configure the control points with the rigid continuity conditions, but set them to generate a surface with approximate continuity. By using approximate G^1 continuity, our data fitting scheme can guarantee a cubic solution, with lower computation price, and result in surfaces with better shape property.

We designed a cubic scheme with approximate G^1 continuity that is similar to the Clough-Tocher's. Three micro triangular Bézier patches will be constructed per each data triangle. For the boundary across different data triangles, approximate G^1 continuity is achieved. For two adjacent micro patches inside the data triangle, we used the same construction as Clough-Tocher's, therefore C^1 continuity is established.

Background

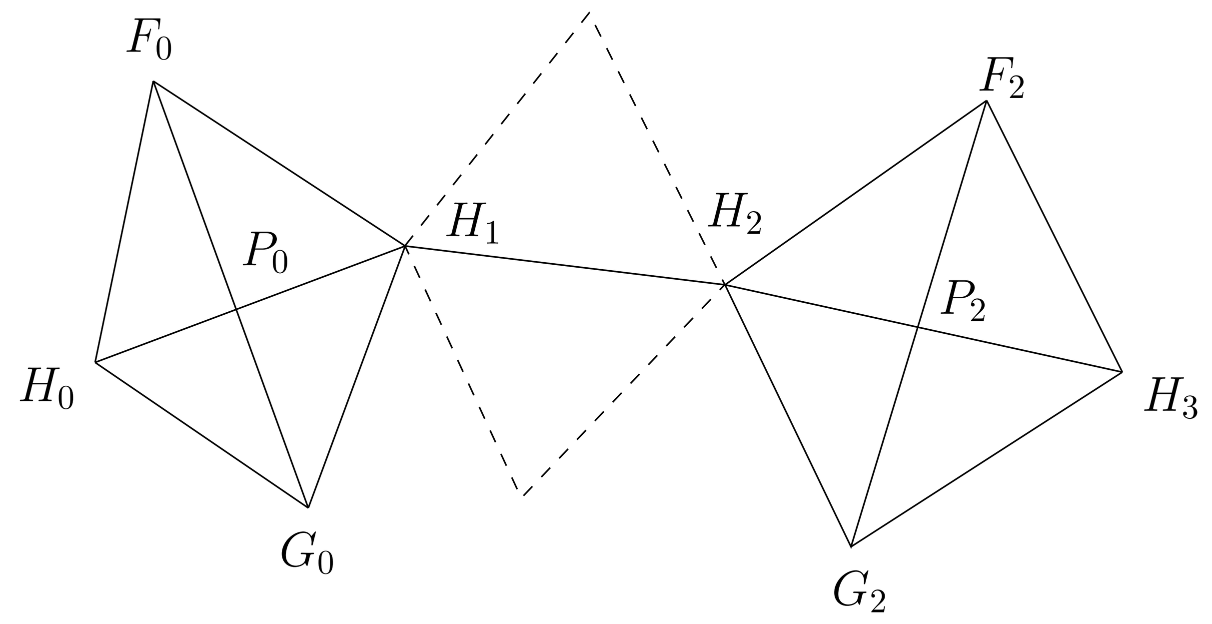
1. G^1 continuity constraints for parametric Bézier patches

   - Continuous tangential plane along the boundary curve.

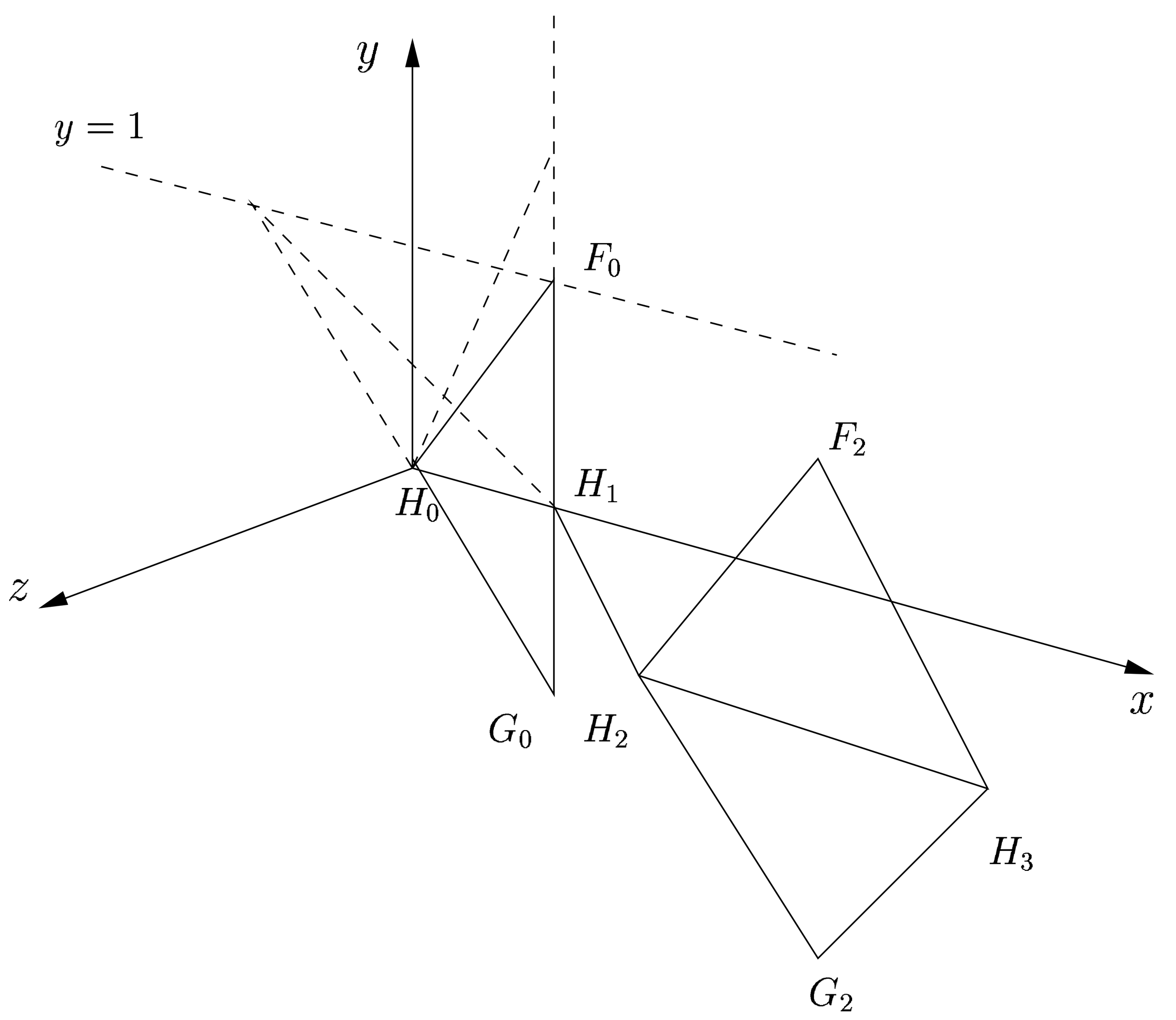


2. Solution with G^1 continuity for cubic surfaces is not always possible.

   - Existence of G^1 solution can be checked by comparing the intersection ratios of side panels.

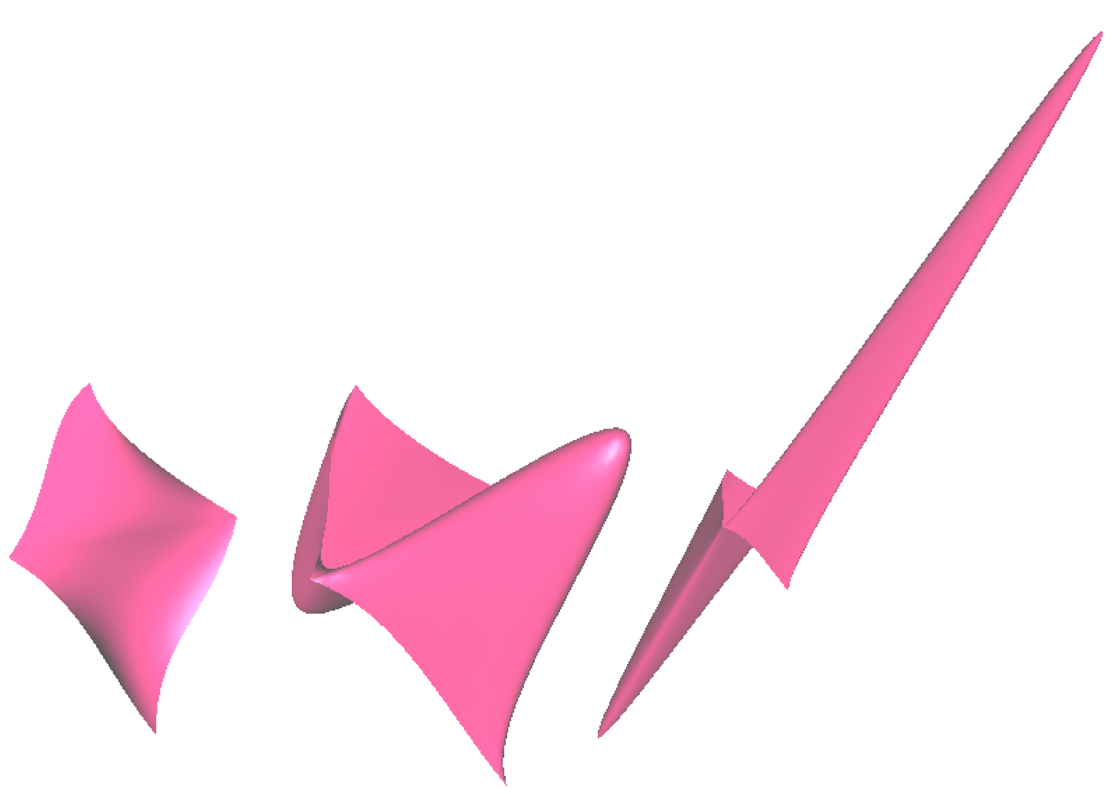


- Piper's example of singularity:
  If control point F_0 moves along $y = 1$, there is no G^1 solution.



3. Close to singularity gives poor shape.

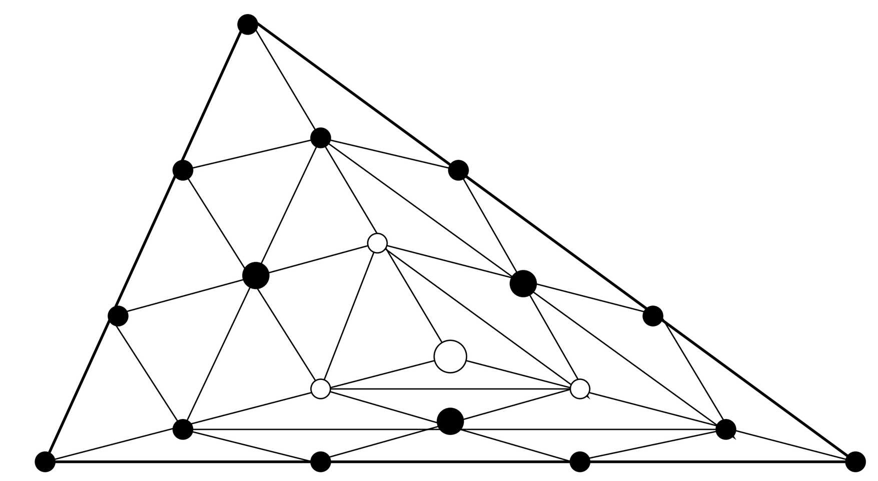
   As F_0 approaches the singular position, surface shape gets worse, the center of the patch will go to infinity.



Solution

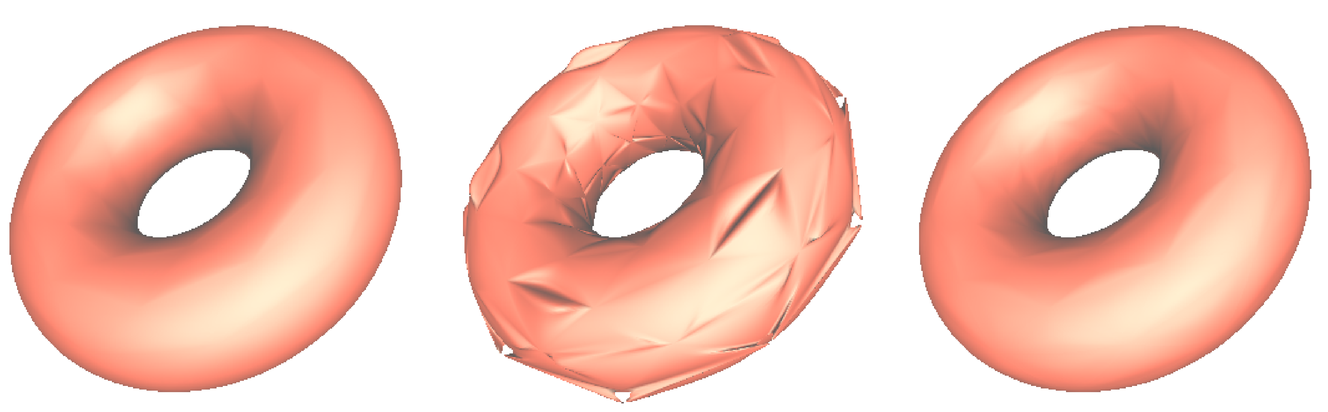
- Approximate continuity:

  Define a surface to be ε-G^1 if the maximum angle between two surface normals at any point along the common boundary is bounded by ε.
- Advantage of using approximate continuity.
  - Solve the problems without precise continuity solution.
  - Improve the shape quality of the resulting surface.
  - Using Clough-Tocher interpolation.



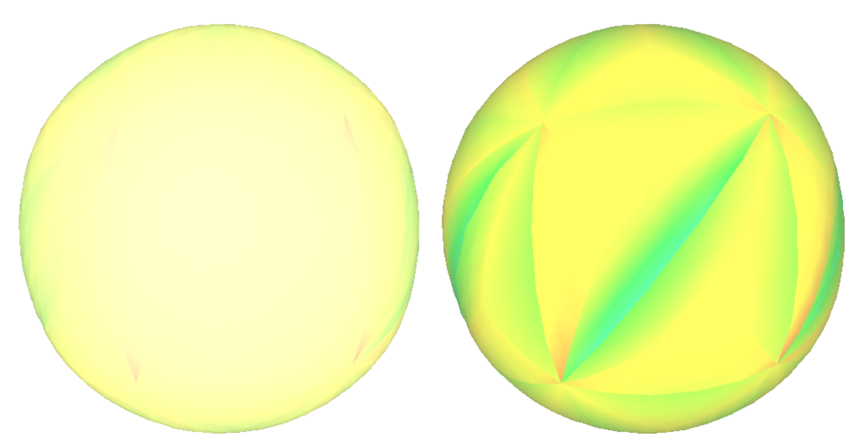
Results

1. Interpolation surfaces using different schemes.



Left: Quartic Shirman-Sequin scheme.
Center: Cubic Clough-Tocher scheme with G^1 continuity.
Right: Cubic approximate G^1 continuity scheme.

2. Curvature analysis.



Left: Quartic Shirman-Sequin scheme.
Right: Cubic approximate G^1 continuity scheme.

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