Approximate Continuity for Parametric, Triangular Bézier Surfaces

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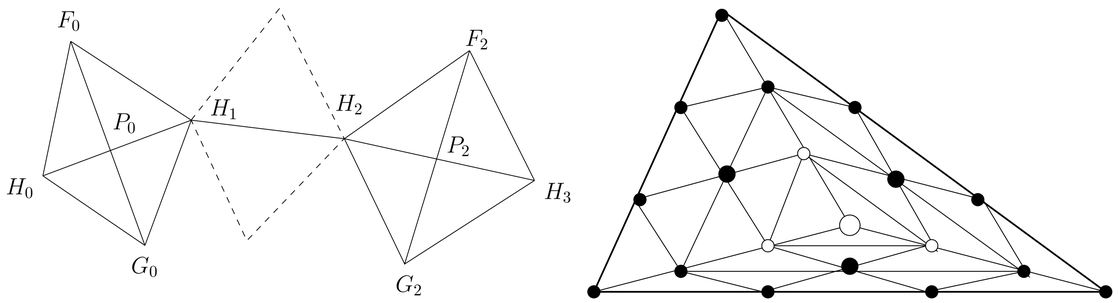
Abstract

For two parametric triangular Bézier patches to meet each other with G^1 continuity, their control points have to fulfill certain constrains. These constraints will lead to high degree patches or bad shape properties. The idea of *approximate continuity* is not to configure the control points with the rigid continuity conditions, but set them to generate a surface with approximate continuity. By using approximate G^1 continuity, our data fitting scheme can guarantee a cubic solution, with lower computation price, and result in surfaces with better shape property.

We designed a cubic scheme with approximate G^1 continuity that is similiar to the Clough-Tocher's. Three micro triangular Bézier patches will be constructed per each data triangle. For the boundary across different data triangles, approximate G^1 continuity is achieved. For two adjacent micro patches inside the data triangle, we used the same construction as Clough-Tocher's, therefore C^1 continuity is established.

G^1 Continuity Conditions

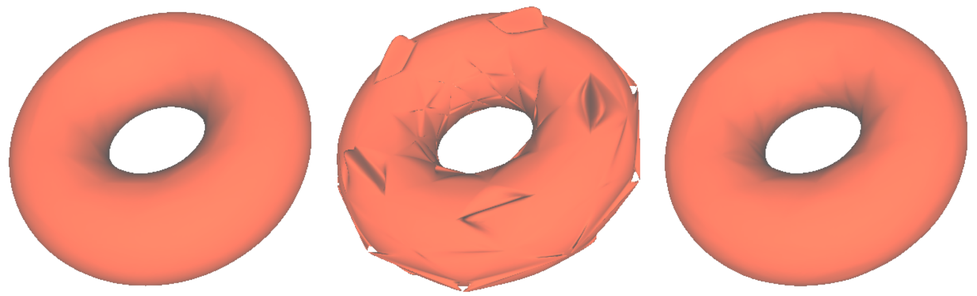


(a) (b)

(a) G^1 Continuity Constraints

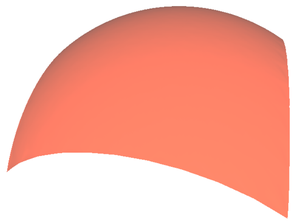
(b) Clough-Tocher Interpolation

Result Surfaces



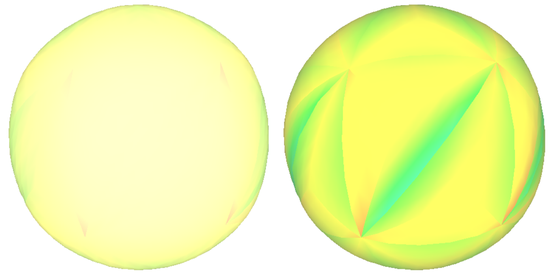
Left: Shirman-Sequin method using quartic patches.
Center: Cubic Clough-Tocher method with G^1 continuity.
Right: Cubic Clough-Tocher method with approximate G^1 continuity.

Results for Singular Cases



Cubic interpolation for Franke function 6 with approximate G^1 continuity, there exists no cubic solution with precise G^1 continuity.

Curvature Analysis



Left: Shirman-Sequin, quartic patches.
Right: Cubic Clough-Tocher method with approximate G^1 continuity.