Tessellating Algebraic Curves and Surfaces Using A-Patches

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Abstract

An implicit surface is defined as the zero set of a scalar function over 3-space. Sign of implicit function determines whether a point is inside or outside the implicit surface:

$$f(P)\begin{cases} = 0 & P \text{ is on the surface.}\\ < 0 & P \text{ is inside the surface.}\\ > 0 & P \text{ is outside the surface.}\end{cases}$$

An algebraic function is an implicit function where the function is polynomial. By using an A-patch representation of algebraic curves and surfaces, we are able to identify regions in which the curve/surface does not lie, and efficiently tessellate the curve/surface in the regions in which it does lie.

A-patches

Bivariate Bernstein polynomials:

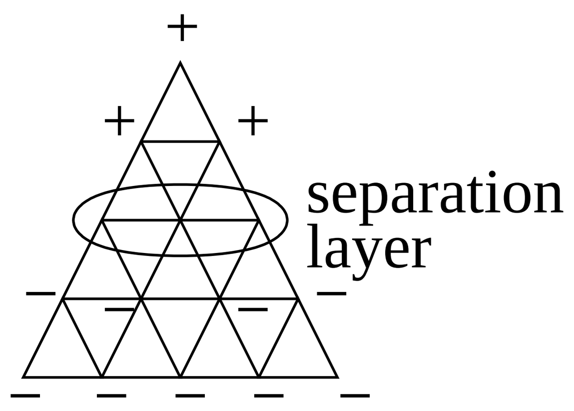
$$B^n_{\vec{i}}(P) = \binom{n}{\vec{i}} p_0^{i_0} p_1^{i_1} p_2^{i_2},$$

where $\vec{i} = (i_0, i_1, i_2)$ with $i_0, i_1, i_2, \geq n$ and $i_0 + i_1 + i_2 = n$, and where (p_0, p_1, p_2) are the Barycentric coordinates.

An A-patch weights scalar values with Bernstein basis:

$$F(P) = \sum_{\vec{i}} c_i B^n_{\vec{i}}(P).$$

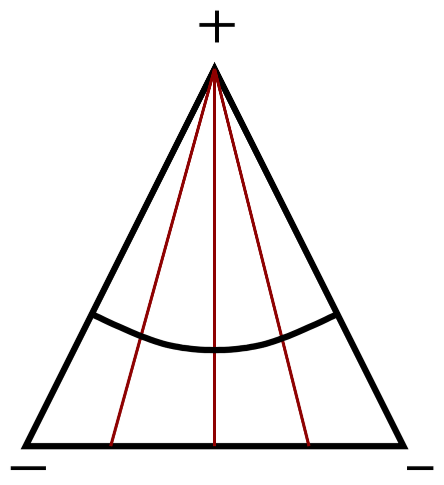
A-patches coefficients separated by sign:



Advantages of A-patches

- Separation guarantees that a single sheet of the algebraic curve passes through triangle.
- Single sheet give means of tessellating:

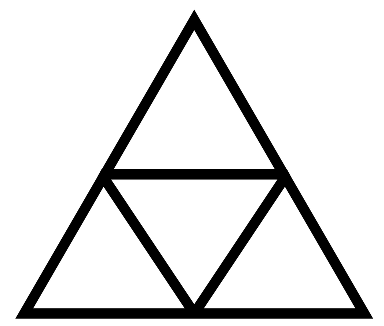
  Root find along lines from corner to opposite edge



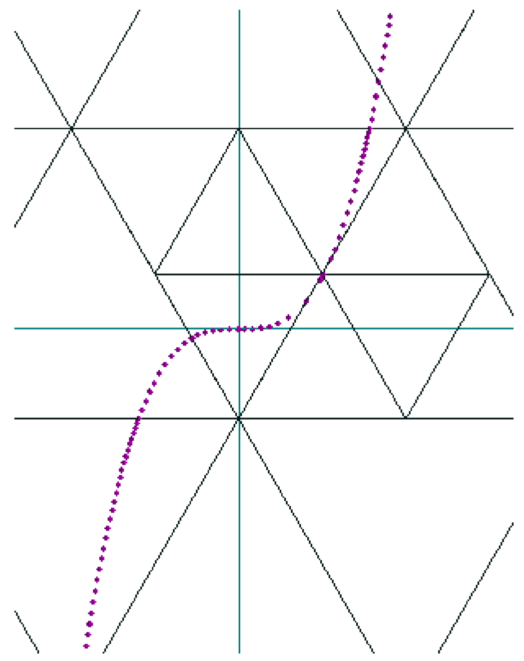
Curves

These observations lead to the following algorithm for tessellating algebraic surfaces:

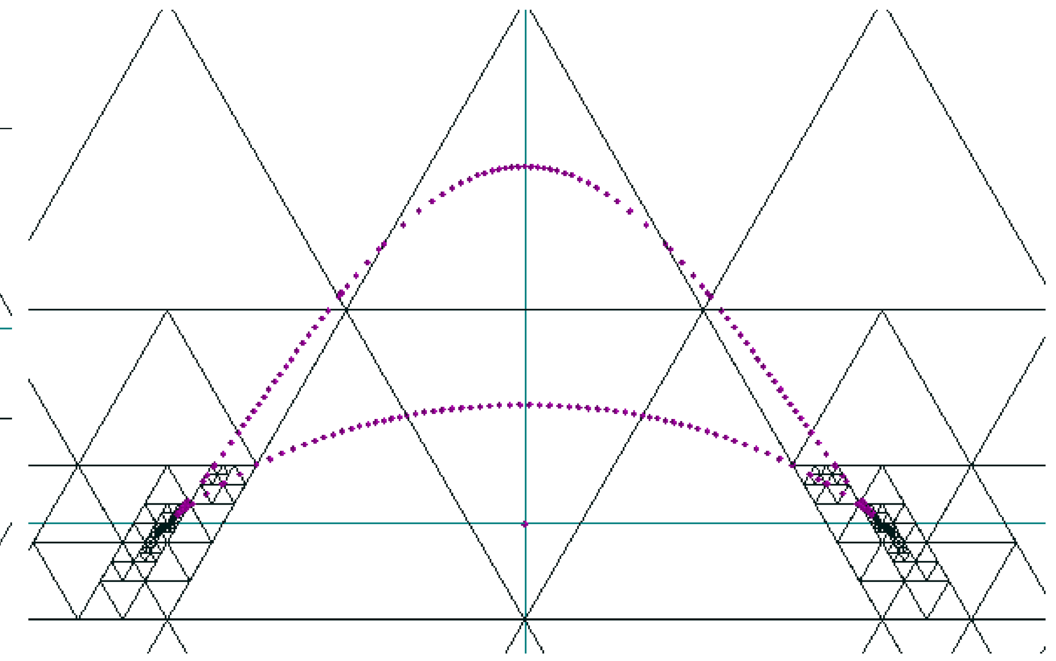
- Make an initial triangular over the region of interest.
- For each triangle,
  - Convert the algebraic to Bernstein-Bézier representation for that triangle.
  - Test the coefficients:
    * If all of same sign, curve/surface does not pass through
    * If in A-patch format, tessellate
    * Otherwise, subdivide the triangle and repeat the process.



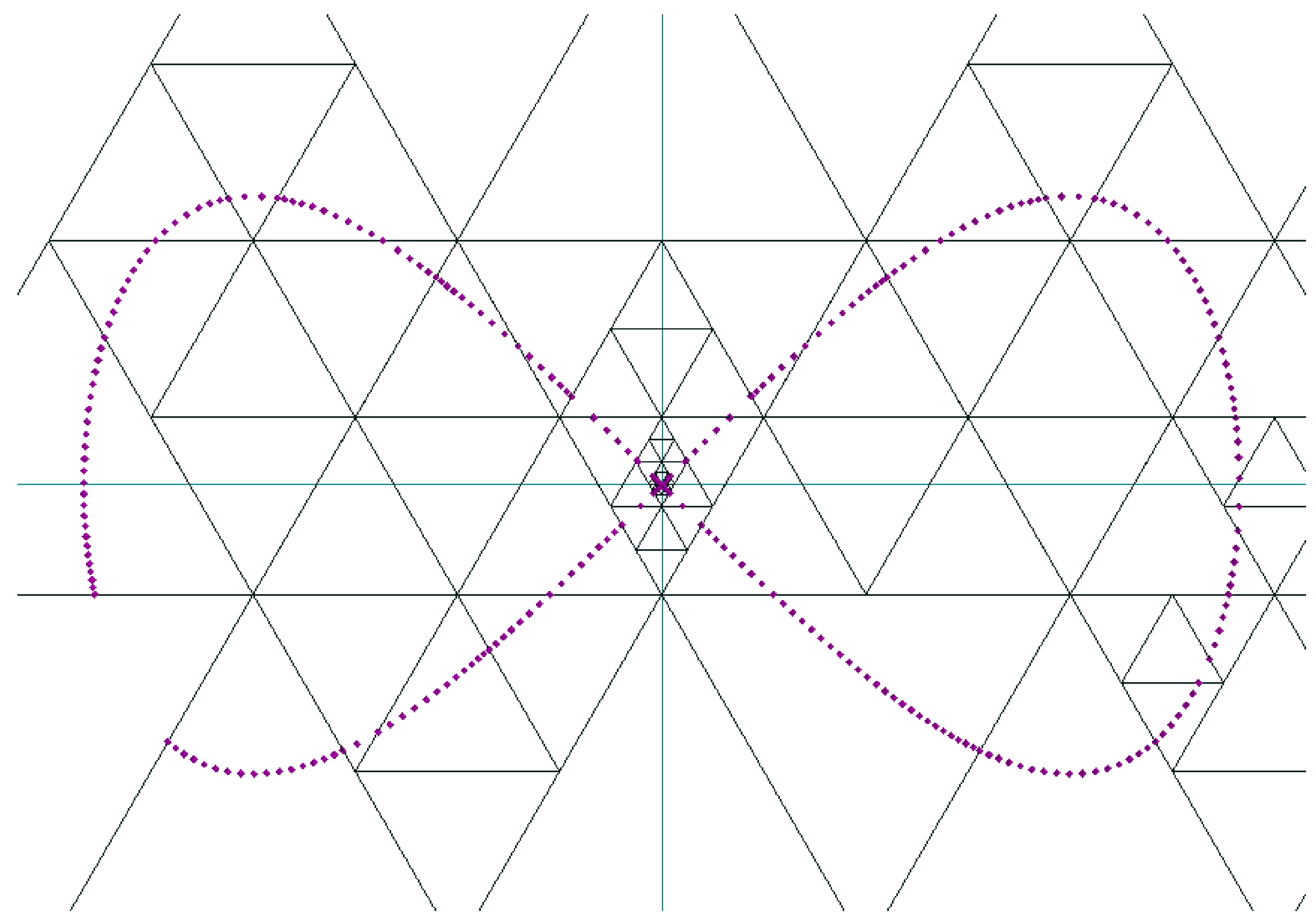
Examples



$10x^2 - y = 0$



$x^4 + x^2y^2 + 4x^2y - 2x^2 + 3y^2 - 4y + 1 = 0$



$x^4 - 2x^2 + 2y^2 = 0$

Surfaces

Idea for surfaces is similar:

- Tetrahedralize space
- Bernstein form of algebra within each tetrahedron
  - All positive/negative, no surface
  - A-patch format: tessellate
  - Mixed coefficients: 12-to-1 subdivision