Better Pasting Through Quasi-Interpolation

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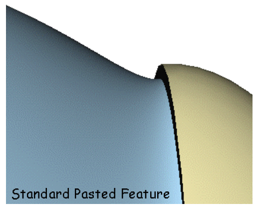
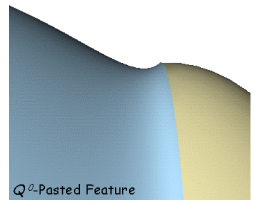
Results

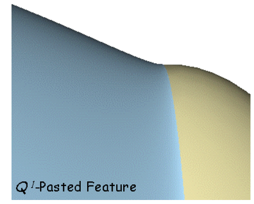
Pasting Method	Mean Position Difference	Max. Position Difference	Mean Normal Difference	# Control Points	Pasting Cost
Standard	5.392e-03	1.572e-02	7.198e-03	81	3402
Q0	2.581e-03	2.570e-03	2.341e-01	81	3822
Q1	1.413e-04	1.864e-03	4.842e-05	81	3030
Std, 1 Refine	1.543e-03	4.550e-03	1.670e-03	225	9450
Std, 2 Refines	4.089e-04	1.149e-03	3.847e-04	729	30618
Std, 3 Refines	1.049e-04	2.891e-04	9.641e-05	2061	109242



Standard Pasted Feature



Q^0-Pasted Feature



Q^1-Pasted Feature

Background

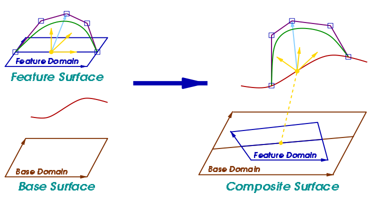
1. Hierarchical Modeling

- model smooth surfaces with local detail
- examples: car door, face
- allows multi-resolution editing

	Hierarchical	Inexpensive	Library	Flexible Paradigm	Guaranteed Continuity
Knot Insertion	–	–	–	–	+
Hierarchical B-Splines	+	+	–	–	+
Displacement Maps	+	–	+	+	+
Surface Pasting	+	+	+	+	–

2. Surface Pasting

- represent each feature control point as a displacement vector
- map feature domain into base domain
- find local coordinate frame on base surface
- map displacement vector to place control point



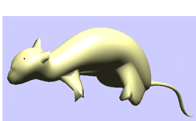
3. Features of Surface Pasting

- developed by Bartels & Forsey
- + computationally inexpensive — only feature control points are mapped
- + pasted feature may have non-rectangular domain
- + flexible modeling paradigm — features may be translated, rotated and scaled
- + library of features to apply to any base
- + hierarchical pasting (hierarchical modeling)
- – only approximates displacement maps — no continuity between feature and base

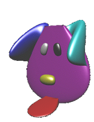
4. Pasted Surfaces

dog model by Leith Kin Yip Chan

turtle model by Selina Siu



model of Sprite the Ferret by Selina Siu

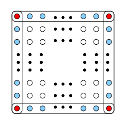


dog model by Clara Tsang

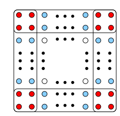
Problem: expensive to reduce C^0 and C^1 discontinuity

5. Feature Boundaries (Standard Pasting)

- no way to eliminate feature-base discontinuities
- feature control point with 0 displacement rests on base
- outer ring has 0 displacement ⇒ approximate C^0
- outer two rings have 0 displacement ⇒ approximate C^1
- knot insertion in feature can reduce discontinuity
  - may need many knots (control points)
  - expensive: each control point must be displaced



approximate C^0 boundary points



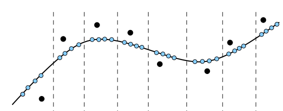
approximate C^1 boundary points

Solution: use Quasi-Interpolation to improve approximation

6. Lyche-Schumaker Quasi-Interpolants

- degree m approximation Qf to a curve f
- each control point of Qf is a weighted sum of linear functionals applied to f:

$$CP_i = \sum_{j=0}^{m} \alpha_{i,j} \lambda_{i,j} f \qquad (1)$$



- $\lambda_{i,j} f = [\tau_{i,0}, \tau_{i,1}, \dots, \tau_{i,j}] f$
- $\boldsymbol{\alpha}_{i,j}$ is a blossom of $p_{i,j}(u) \equiv (u - \boldsymbol{\tau}_{i,0})(u - \boldsymbol{\tau}_{i,1}) \cdots (u - \boldsymbol{\tau}_{i,j-1})$
- $Qf = f$ when f is a degree m or lower polynomial
- otherwise, the approximation error has the best possible order

7. Modified Functionals, Coefficients

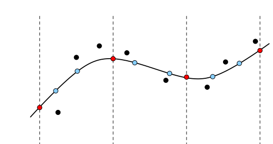
- Lyche-Schumaker quasi-interpolant uses cheap coefficients and expensive linear functionals
- for pasting, linear functionals are recalculated frequently, coefficients less so
- we made new cheaper $\lambda_{i,j} f = f^{(r)}(\tau_{i,j})$
- results in more expensive coefficients based on blossom of $p_{i,j}(u) = \prod_{k \neq j} \frac{u - \tau_{i,k}}{\tau_{i,j} - \tau_{i,k}}$

8. Our Q^d Operators

- modify Q to reproduce position and d derivatives at its endpoints
- linear functionals for control points near ends are derivatives of original curve at endpoints
- equation (1) suggests $m+1$ samples per control point
- we use a new sampling discipline to reduce the number of samples per control point

9. Sampling Discipline

- divide control points into groups of about "degree"
- choose intervals to sample from
  - interval endpoints are average of Greville points of adjacent control points
- sample uniformly within intervals
- share first & last samples
- sharing gives about one base sample per control point



each black CP is weighted average of blue & red samples in group

10. Quasi-Interpolated Surface Pasting

- use quasi-interpolation to set control points around feature boundary
- treat each edge as a separate curve approximation problem
- corner points are shared between two edges
- use Q^d operator to set $d+1$ outermost rings of control points
  - Q^d operator was constructed so corner control points are set consistently
  - gives improved approximate C^d continuity around feature boundary