Self-Organized Criticality as a Method of Procedural Modeling

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Overview

Self-organized criticality (SOC) is a theory of fractal dynamics in which a physical system approaches an attractor state that is scale-free. In the simple example of a sandpile, the grains of sand can slip down steep slopes and the resulting changes to the system propagate via avalanches. By the time the sandpile reaches the attractor state, the avalanches have propagated throughout the entire system and the features of the resulting landscape are not tied to a specific length scale.

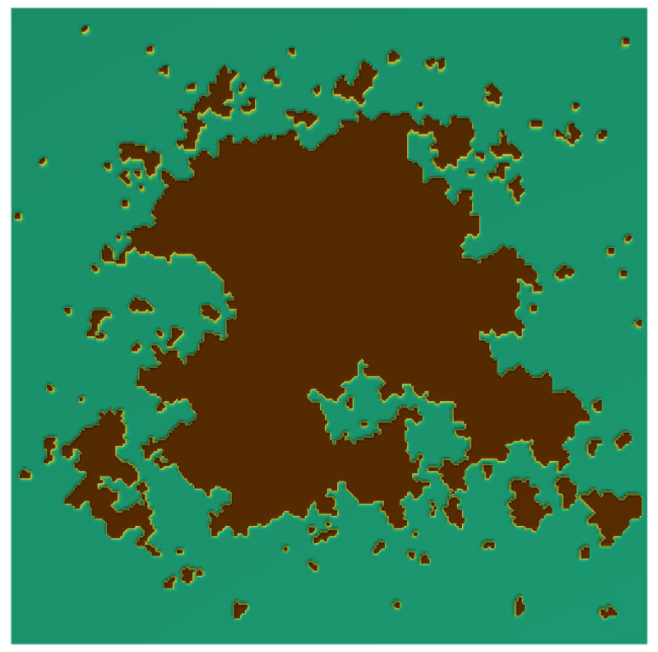
The elements of SOC system evolution can be used for procedural creation of geometry. The main benefits of the approach are formulation in terms of the dynamics of the underlying system and use of avalanches as a modeling paradigm. Avalanches link local and global shape and help avoid creasing artifacts associated with scaling issues.

Basic Coastline Model

Sapoval et al. propose the following SOC model of coastline erosion [4]. There is a force F acting everywhere along the coast. Pieces of the coast erode based on their intrinsic resistance R and degree of exposure. As the coast erodes, its perimeter L becomes longer which attenuates F according to the formula:

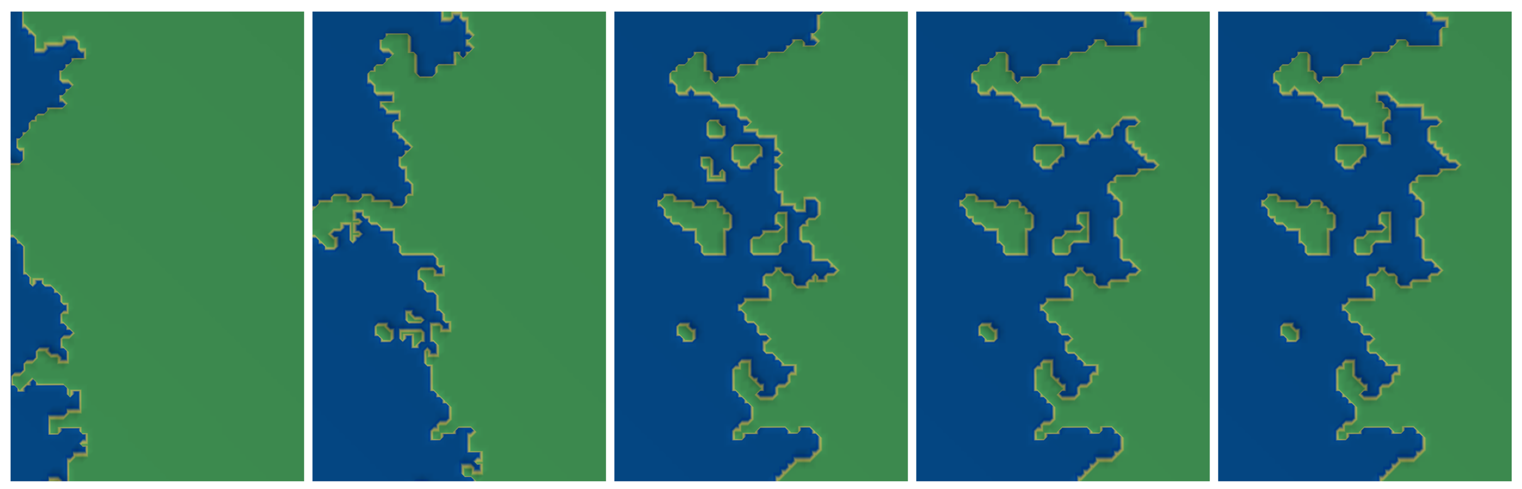
$$F(t) = \frac{f}{1 + \frac{gL(t)}{L(0)}}$$

The parameters controlling the shape of the coast are constants f and g, as well as the distribution of R. The resulting shape of the coast is a statistically self-similar fractal and is an attractor state of the model.



Coastline produced with SOC model of Sapoval et al.

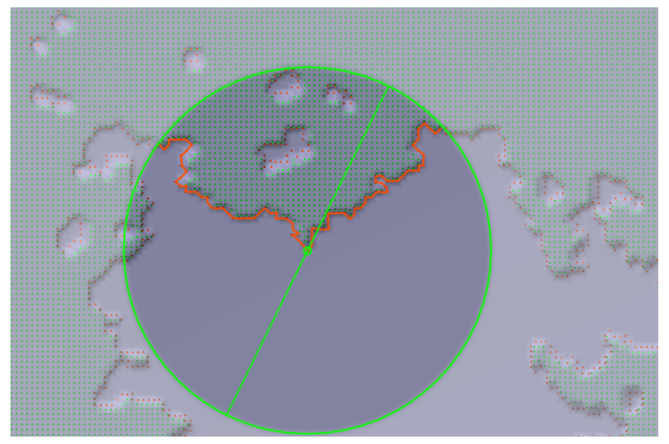
Avalanches can start when an eroding piece of the coast exposes its neighbors to further erosion. Besides the initial conditions, the size of the avalanches depends on the current $F(t)$, which in turn depends on the global perimeter.



Coastline evolution is dominated by avalanches.

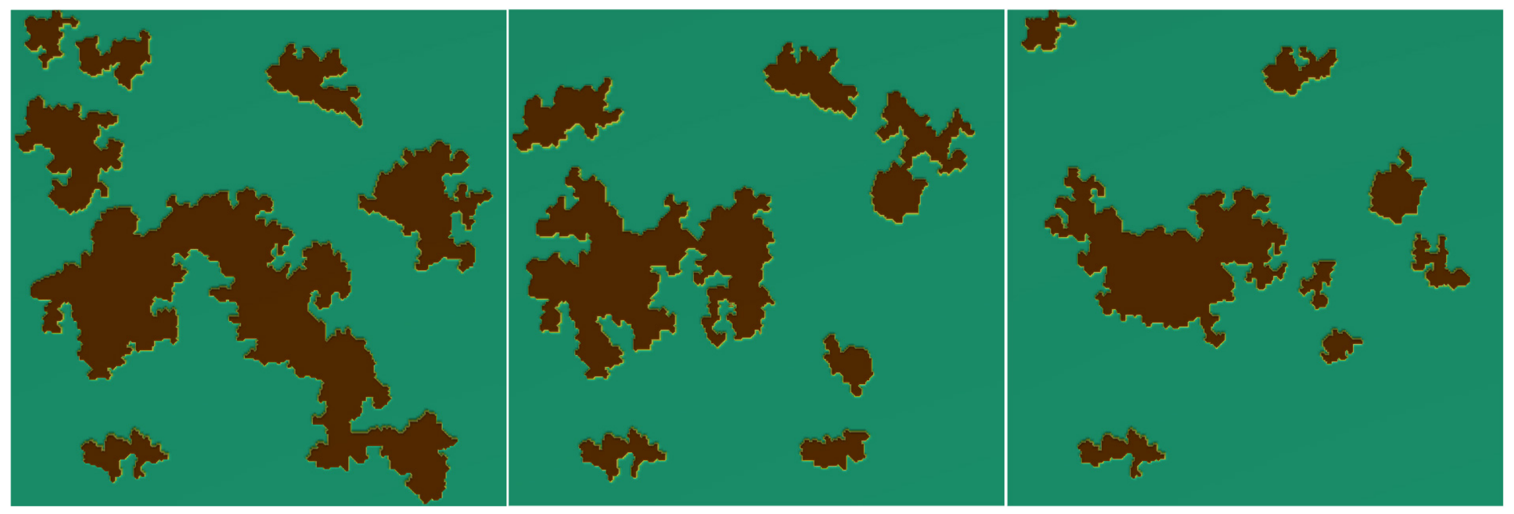
Local and Global Shape

Avalanches are a useful modeling paradigm because of their link to dynamics and the natural way in which they distribute shape changes. However, when it is desirable to create variation between local and global features, it is necessary to re-introduce a notion of scale into how the avalanches behave. In the case of coastlines, my extension to the model is a local formulation of the perimeter.

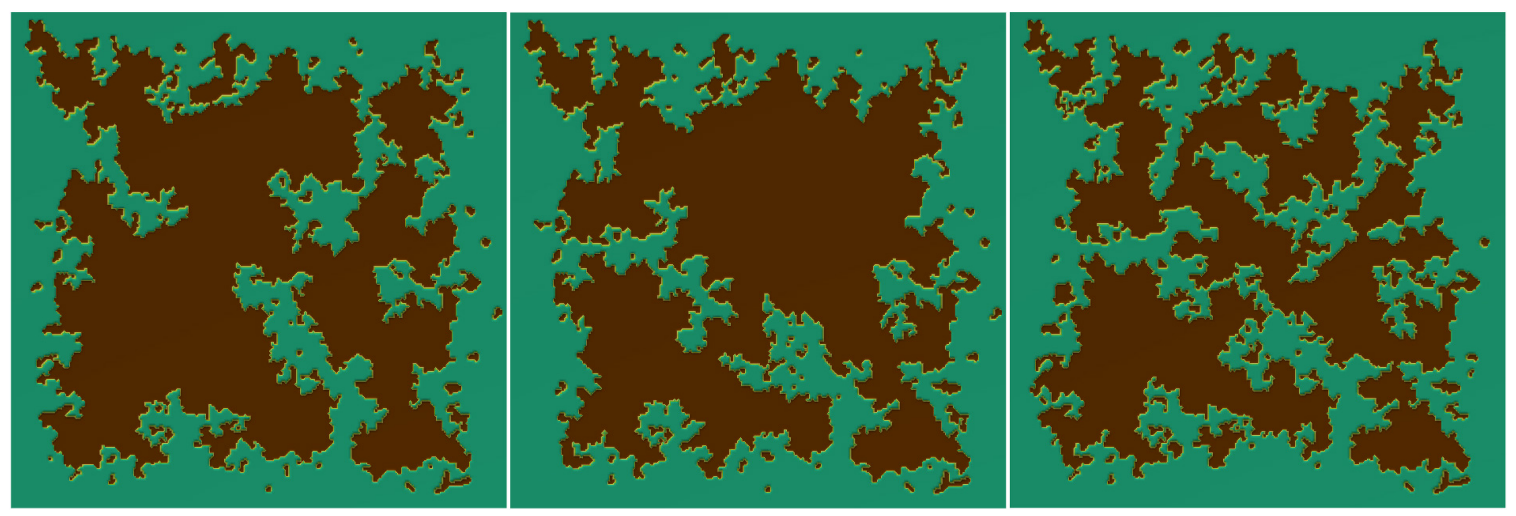


Localization of character.

Eroded and uneroded locations are represented by nodes in a graph. The nodes c_i on the boundary of the uneroded area form the coastline. The force of erosion is different at each c_i: $F(c_i, t)$. The distance method of calculating $F(c_i, t)$ is to compare the length of the coast within the search radius R to $2R$. This doesn't include the perimeter of small islands lying off the coast. (In the following images R decreases from left to right.)



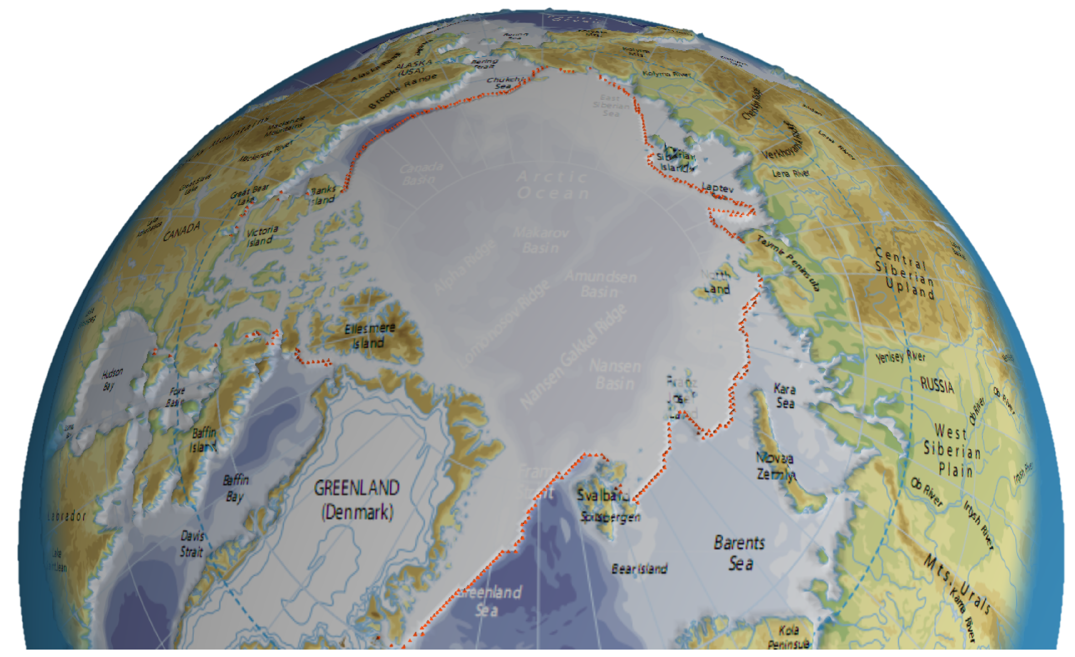
The counting method of calculating $F(c_i, t)$ is to count the number of c_j lying within the search radius and compare it to the number that would be expected if the coastline was a straight line (the number depends on the sampling density of the graph).



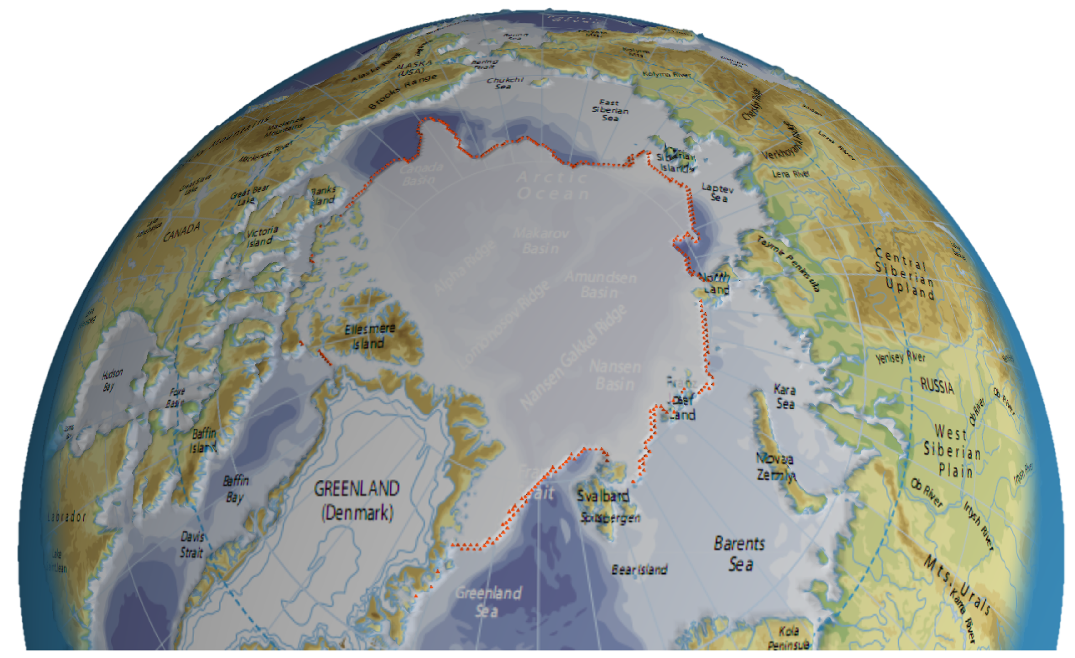
Polar Ice Cap

The extended SOC-based procedural modeling method can simulate the melting of the polar ice cap in a plausible way. I have used the following parameters: distribution of land [3], distribution of ice in 1982 [1], approximate ice thickness [2], and latitude. The erosion avalanches depend on the distance method of calculating the local perimeter.

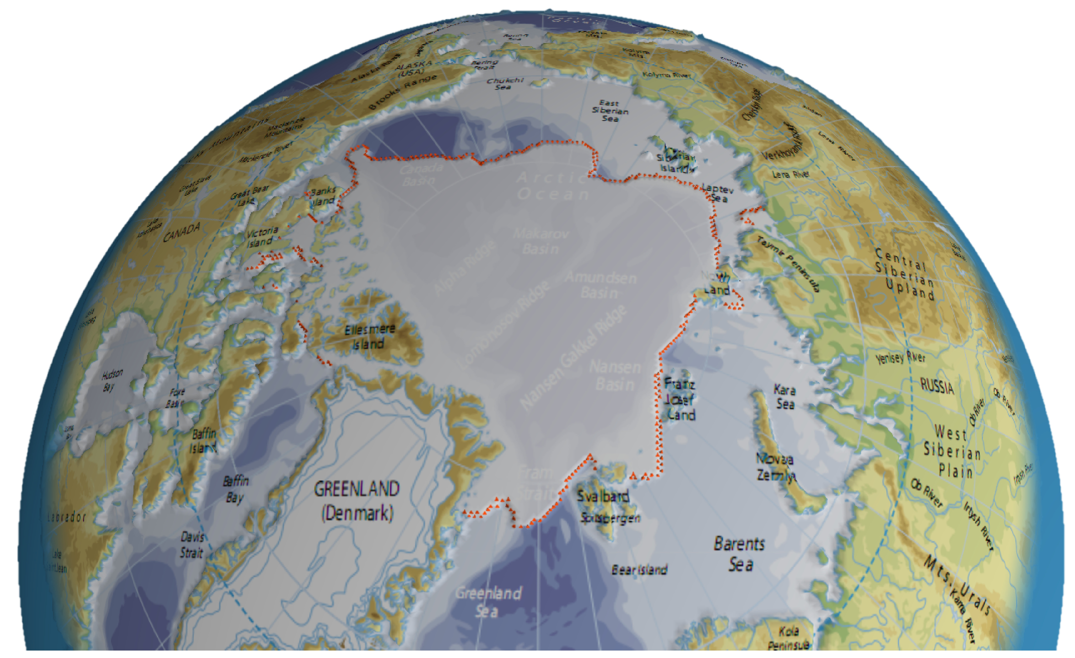
The result of the simulation is a distribution of ice that looks similar to the actual distribution in 2008 [1].



Polar ice cap in 1982.



Simulation of ice cap melting.



Polar ice cap in 2008.

[1] H. Ahlenius. *Arctic Sea Ice Minimum Extent in September 1982 and 2008*. UNEP/GRID-Arendal, 2007. http://maps.grida.no/go/graphic/arctic-sea-ice-minimum-extent-in-september-1982-and-2008

[2] J. Maslanik and C. Fowler. *Arctic Sea Ice Age*. University of Colorado, 2009.

[3] P. Rekacewicz. *Arctic, Topography and Bathymetry*. UNEP/GRID-Arendal, 2005. http://maps.grida.no/go/graphic/arctic-topography-and-bathymetry

[4] B. Sapoval, A. Baldassarri, and A. Gabrielli. Self-Stabilized Fractality of Seacoasts through Damped Erosion. *Physical Review Letters*, 93(9), 2004.