Self-Organized Approach to Modeling Hydraulic Erosion Features

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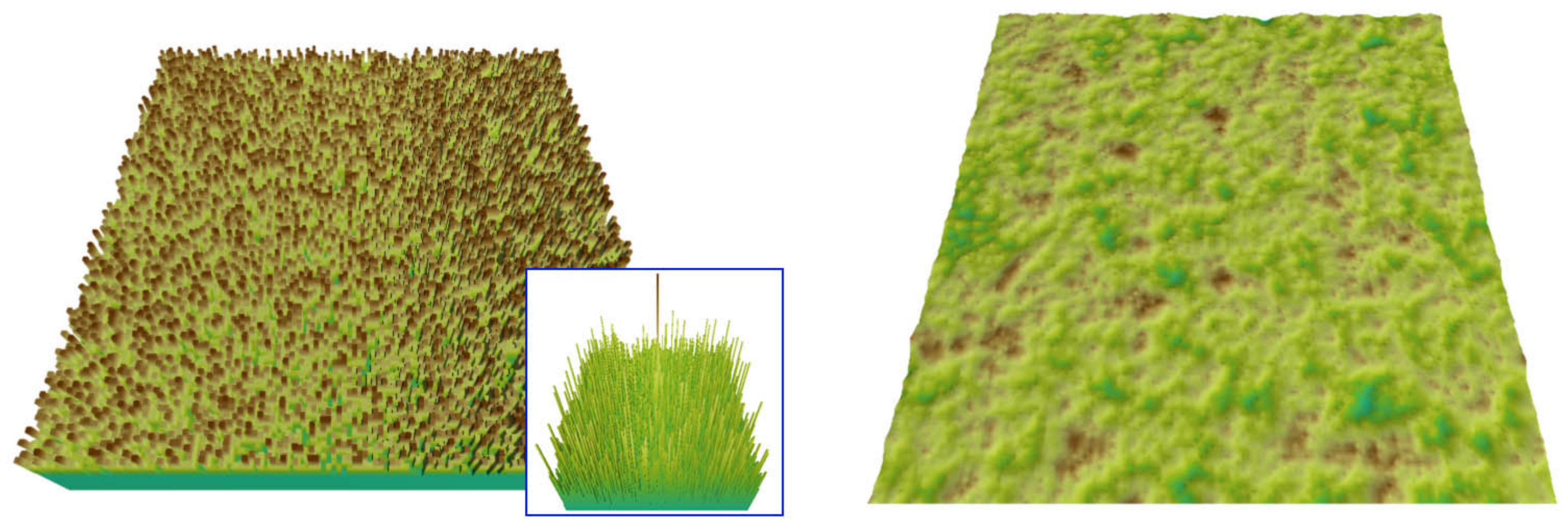
ABSTRACT

Landforms affected by hydraulic erosion exhibit emergent features, such as channels that organize into networks through tributary capture. Our procedural modeling method attempts to simulate these features realistically by using a variant of a principle followed by many physical self-organized systems. The general nature of the approach makes it applicable to modeling river-like channels on top of terrains and cave-like channels inside of volumes of porous rock.

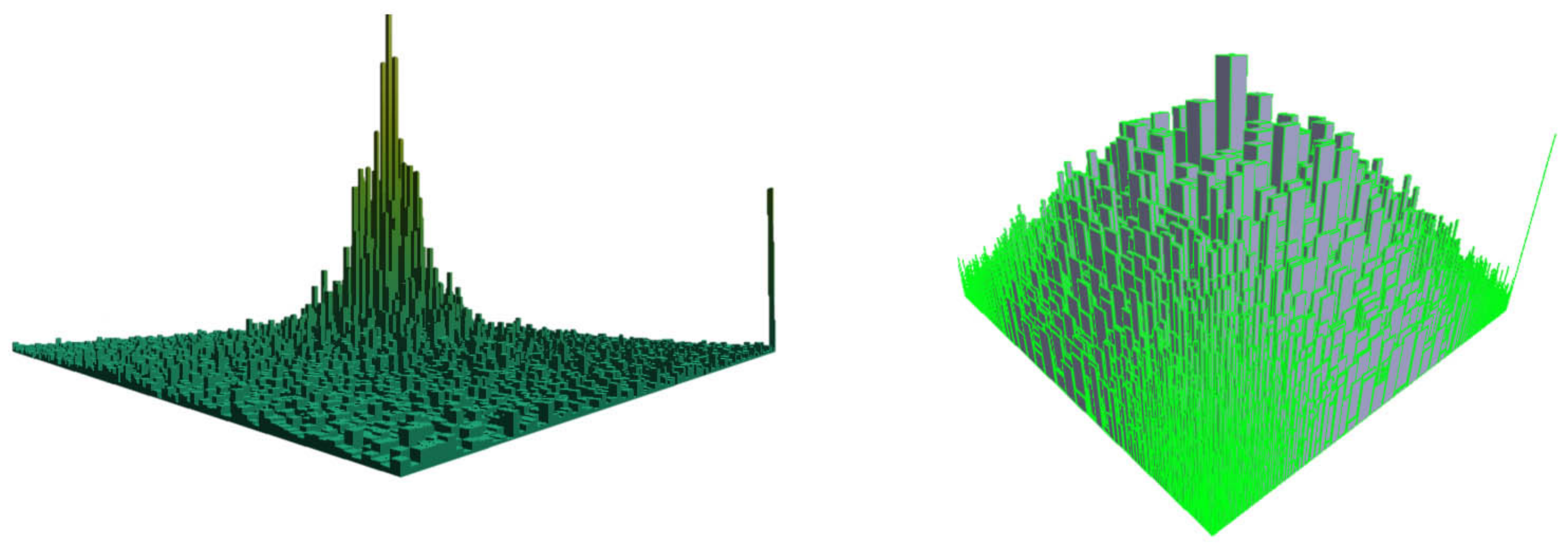
AVALANCHING

Rodríguez-Iturbe et al. have proposed an explanation for the evolution of river networks in terms of self-organized criticality [3], a theory of fractal dynamics in which a physical system approaches a scale-free attractor state. During this process the system undergoes avalanches of change that distribute detail throughout the entire system.

We have found that utilizing avalanching separately from its parent theory makes it possible to re-introduce scale into their dynamics. This way the produced objects are not restricted to having a scale-free scaling character. In the following example avalanches transport material based on a slope constraint, smoothing the initial terrain, which is uniformly rough.



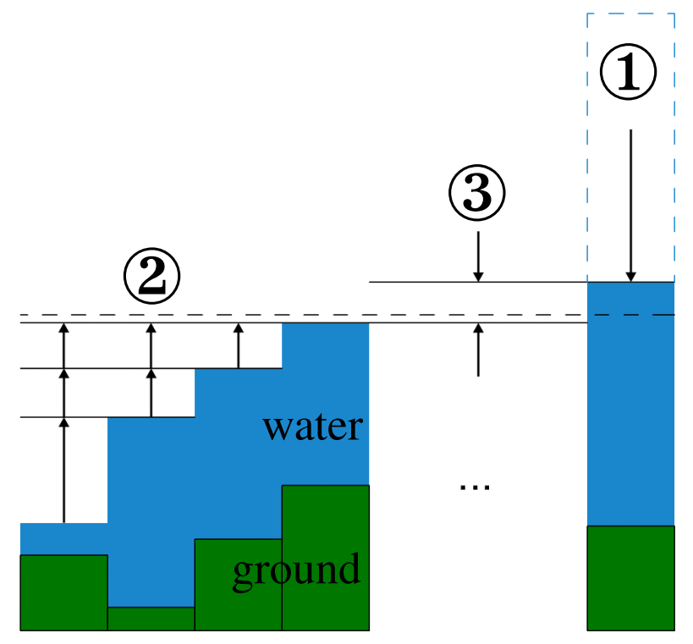
The scaling character can be judged using spectral density $S(f)$, which is a way to visualize the distribution of frequencies in a signal. The spectral density of the resulting terrain falls off approximately as $\frac{1}{f^{\beta}}$, which can be confirmed via a triple-log plot that turns the falloff into a linear one.



The $\frac{1}{f^{\beta}}$ falloff corresponds to self-affine scaling, which is appropriate for modeling hydraulic erosion features. For example, rivers have been found to follow several different power laws in respect to drainage area and bifurcation, which make them self-affine fractals. In general, a terrain dominated by erosion can be expected to be close to a self-affine fractal in shape, because of its unequal scaling behaviour in horizontal and vertical directions.

EROSION SIMULATION

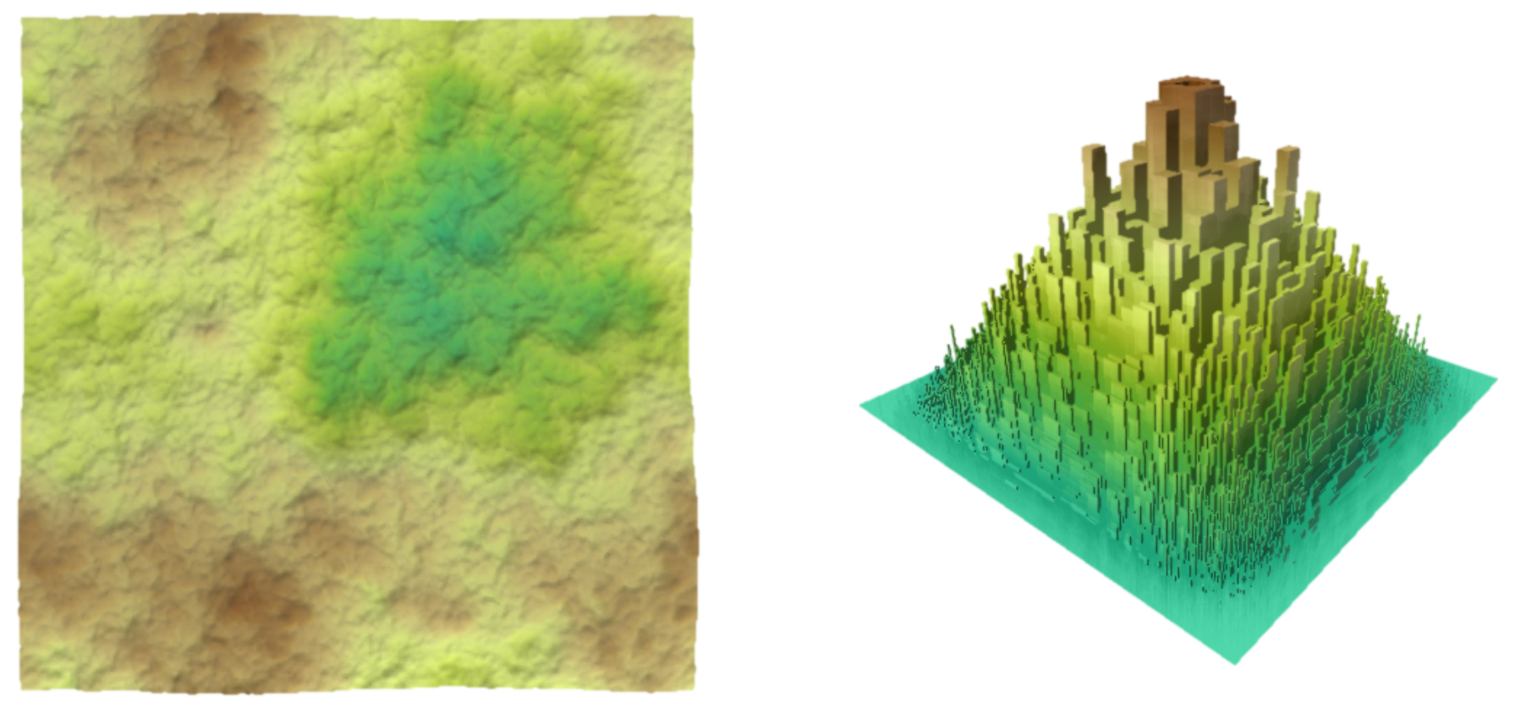
As an abstraction aid, we discretize each type of the problem setting as a graph with nodes representing locations on a terrain in the 2D case and voxels of a porous rock matrix in the 3D case. A key difference is that a voxel can only contain up to one unit of water, as determined by local porosity.



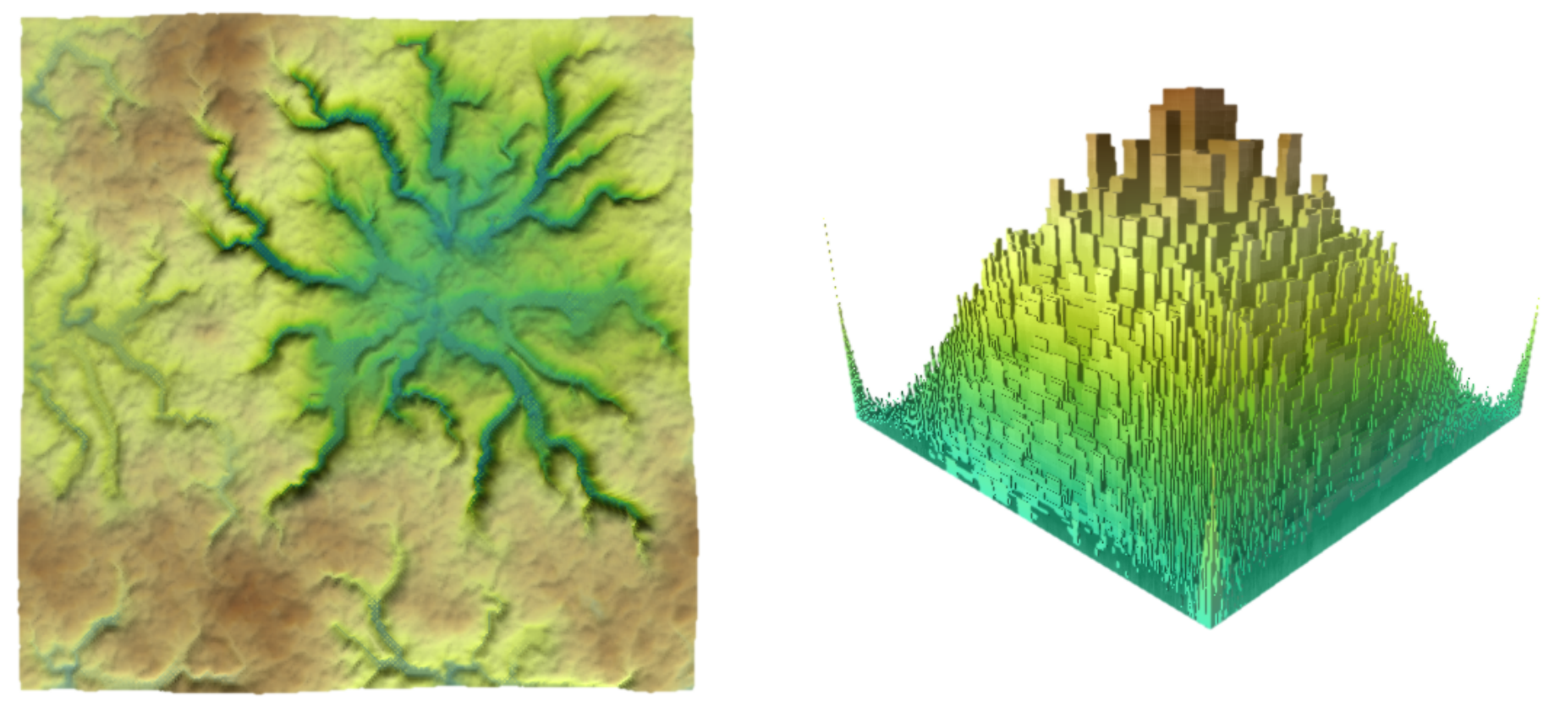
Specialized water column algorithm.

To simulate water flow, we use a water column algorithm that iteratively equalizes the columns of water between neighbouring locations. In some versions of this algorithm the exchange of water is similar to averaging and has a dissipative effect on water flow. To help water organize into channels, our specialized algorithm takes water from a central location ①, sorts the neighbours in ascending order of total combined height, and then attempts to raise the level of water ② in n lowest neighbors to match the total height of $(n+1)$-th neighbour. If there is enough water, the algorithm will eventually equalize all participating locations to a common level ③.

We combine the water simulation with avalanching by eroding material in areas where the local flow of water is large. Doing this sets up erosion avalanches that induce large streams to become larger through tributary capture. Such an avalanching strategy produces features with an appropriate scaling behaviour, as can be seen by applying our simulation to a terrain generated with fractional Brownian motion (fBm), which is a statistically self-affine fractal.



fBm and triple-log plot of its spectral density.

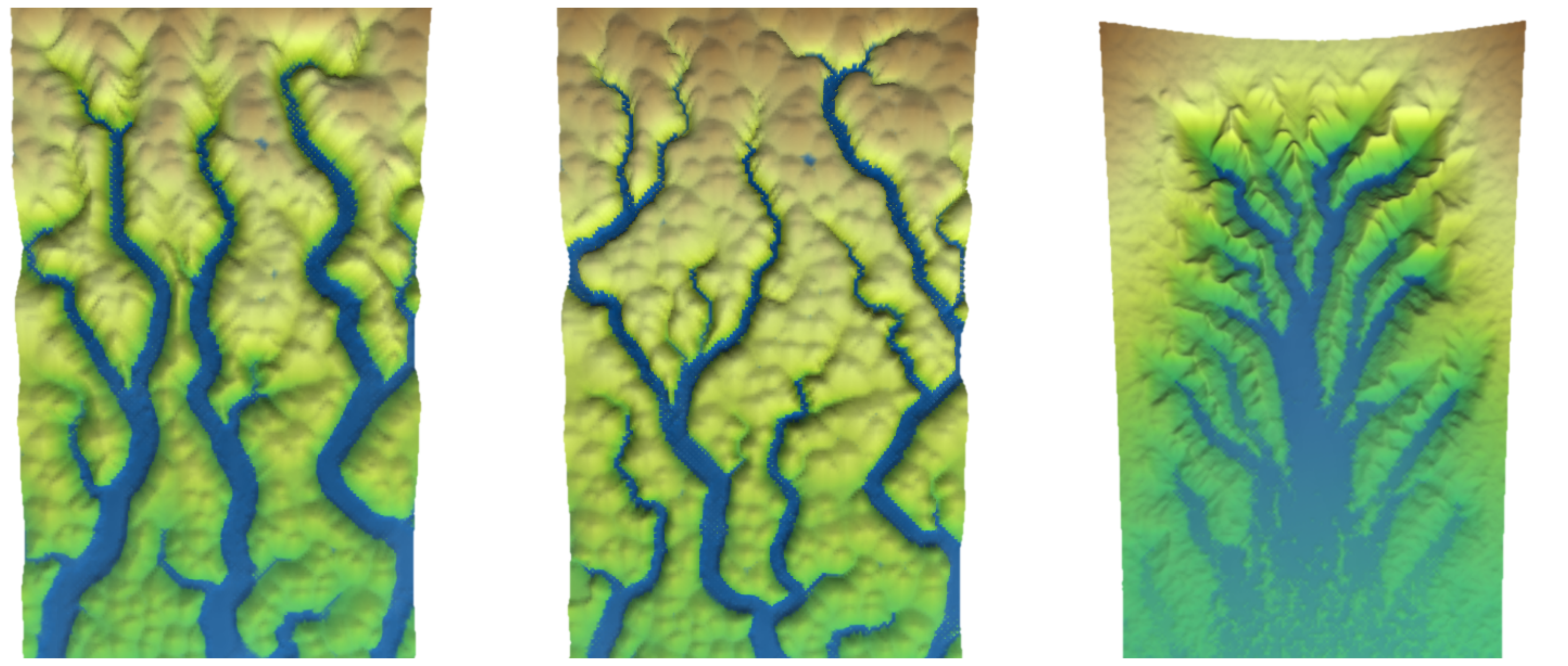


Eroded terrain and resulting spectral density.

After erosion the spectral density of the terrain exhibits the same falloff relationship across most frequencies as the original. The high frequency features (near the edges of the plot) that do not follow the pattern are not significant, because they are too small in respect to the size of the river-like channels.

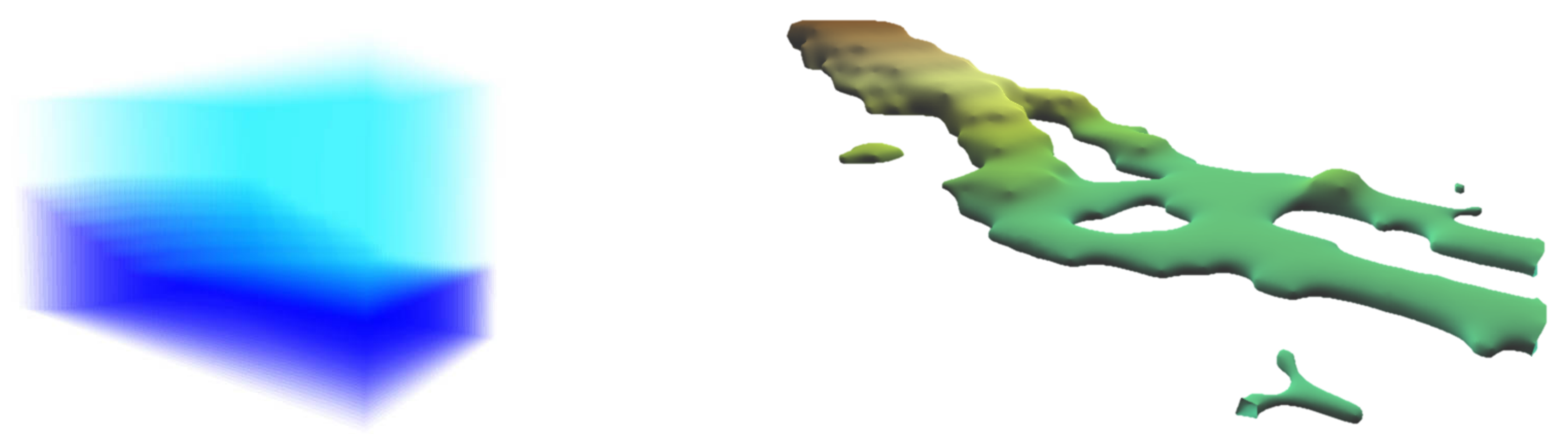
RIVER-LIKE CHANNELS

The following are some examples of river-like erosion features generated with our method. Note that the channels have finite widths, i.e., a confluence of two channels results in a wider one.

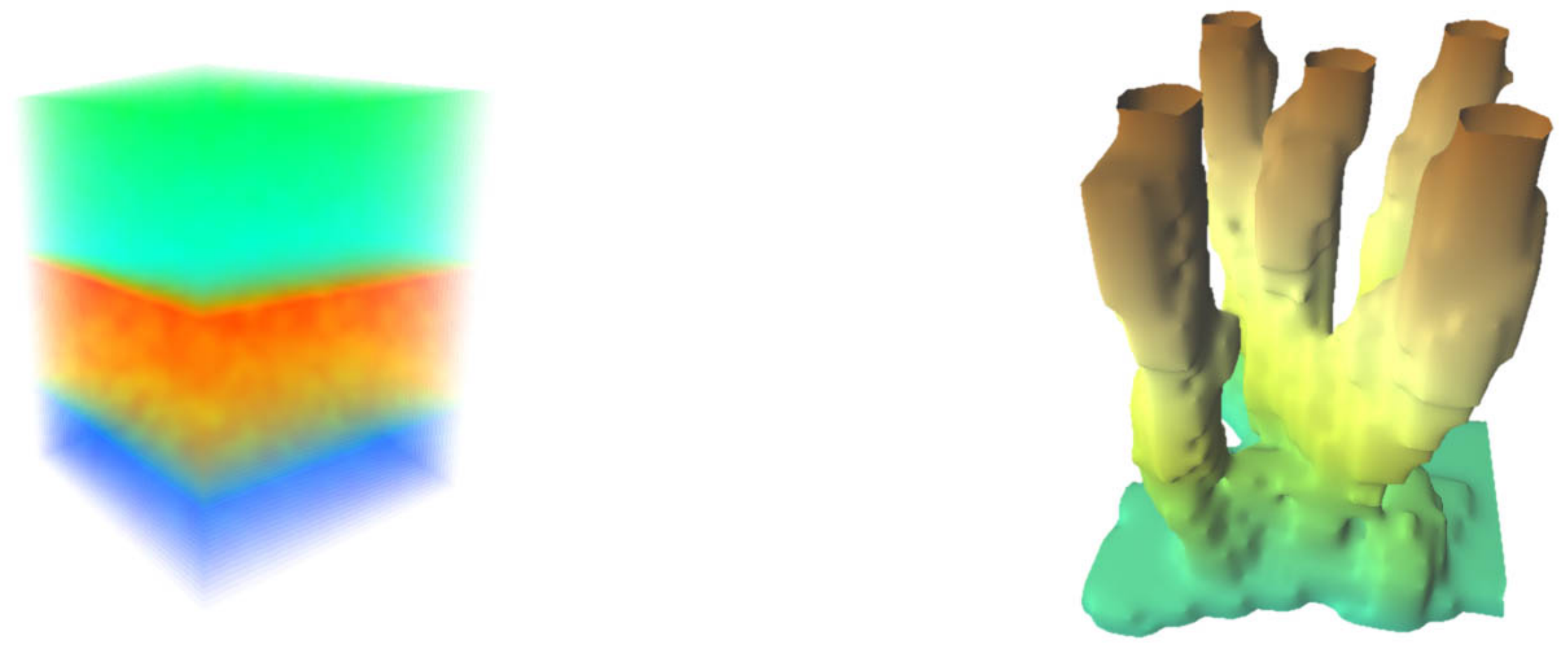


CAVE-LIKE CHANNELS

Water tends to organize into channels when it flows through porous rock underground, just as it does when it flows over a terrain.



In the above example, we apply our algorithm to a rock matrix of variable permeability that contains an impermeable half-ramp half-block. There is one spring that creates a net flow in a roughly horizontal direction.



The second set of initial conditions consists of several layers of rock with different properties and five springs. The net flow is in a vertical direction. Note that some of the conduits link up as tributaries.

[1] M. Laverty. Fractals in Karst. *Earth Surface Processes and Landforms*, 12(5):475–480, 1987.

[2] F. K. Musgrave, C. E. Kolb, and R. S. Mace. The Synthesis and Rendering of Eroded Fractal Terrains. In *SIGGRAPH*, pages 41–50. ACM, 1989.

[3] I. Rodríguez-Iturbe and A. Rinaldo. *Fractal River Basins*. Cambridge University Press, 1997.