Natural Quadrics

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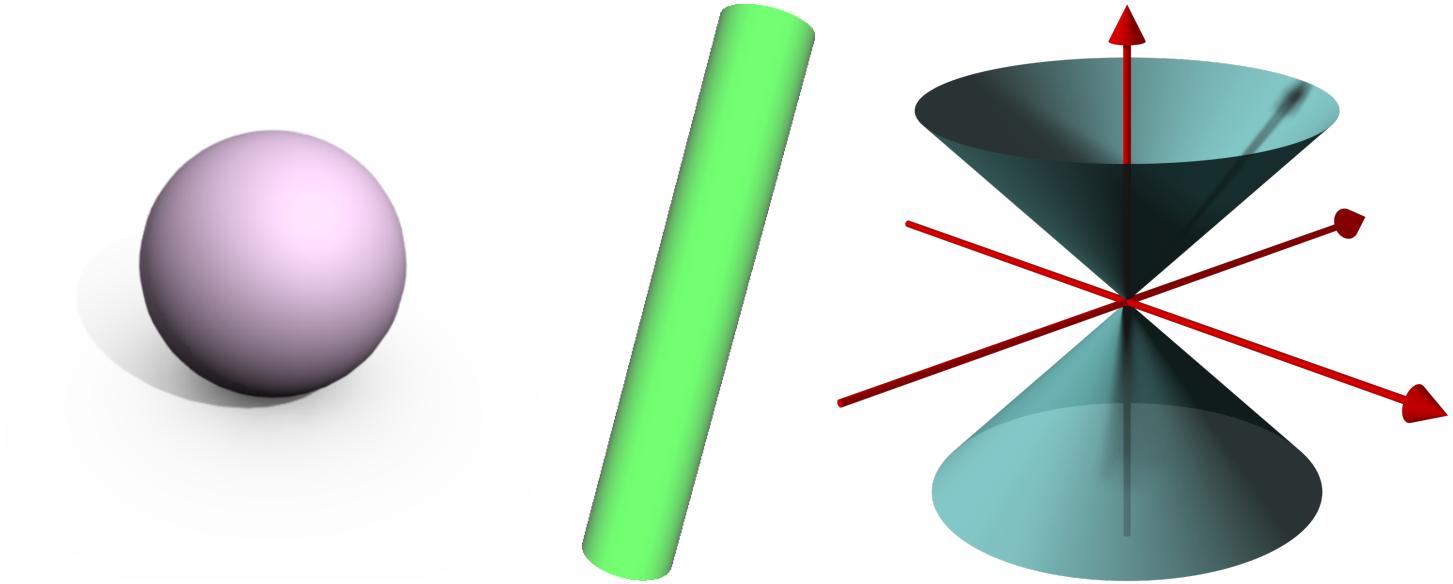


Quadric Surfaces

Degree 2 surface

$$Ax^2 + By^2 + Cz^2 + 2Dxy + 2Eyz + 2Fxz + 2Gx + 2Hy + 2Jz + K = 0$$

Matrix form:

$$PQP^T = 0,$$

$$P = (x, y, z, 1)$$

$$Q = \begin{pmatrix} A & D & F & G \\ D & B & E & H \\ F & E & C & J \\ G & H & J & K \end{pmatrix}$$

Coefficients unintuitive: what to use for a sphere? Cylinder? Cone?

Natural Quadrics

Dual form (parametric, implicit):

- Sphere: center C and radius r:

$$(P - C) \cdot (P - C) - r^2 = 0$$

- Cylinder: center of base B, axis $\vec{v}$, radius r:

$$(P - B) \cdot (P - B) - ((P - B) \cdot \vec{v})^2 - r^2 = 0$$

- Cone: Tip B, axis $\vec{v}$, cone angle α:

$$((P - B) \cdot \vec{v})^2 - cos(\alpha)\,(P - B) \cdot (P - B) = 0$$

- Plane: Point B, normal $\vec{n}$:

$$(P - B) \cdot \vec{n} = 0$$

Ray Tracing

- Substitute ray $r(t) = R + t\vec{w}$ into quadric Q,

  solve for $Q(r(t)) = 0$ for $t > 0$

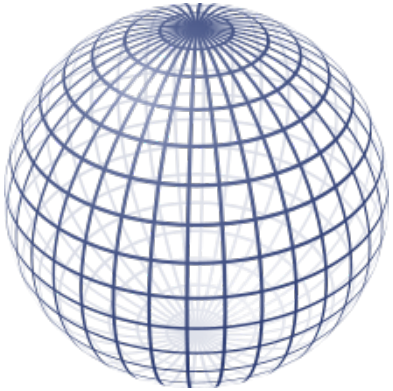
  Quadratic equation

- May want to "cap" cylinder, cone

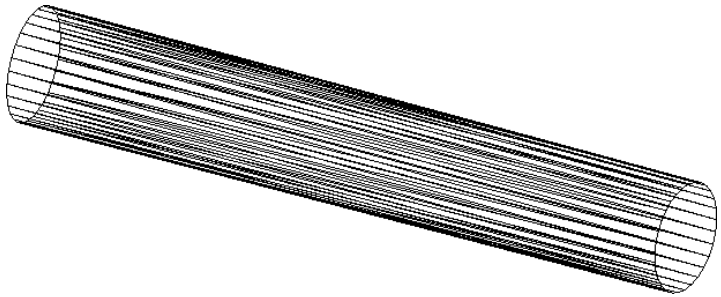
- Can special case code for speed

Tessellation

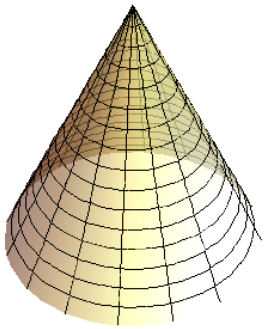
- Dual form (parametric, implicit) means we can tessellate natural quadric without using Marching Cubes, etc.



Sphere



Cylinder



Cone

Uses

- In mechanical design, 85% of parts are natural quadrics