Interpolating and Approximating Implicit Surfaces from Polygon Soup

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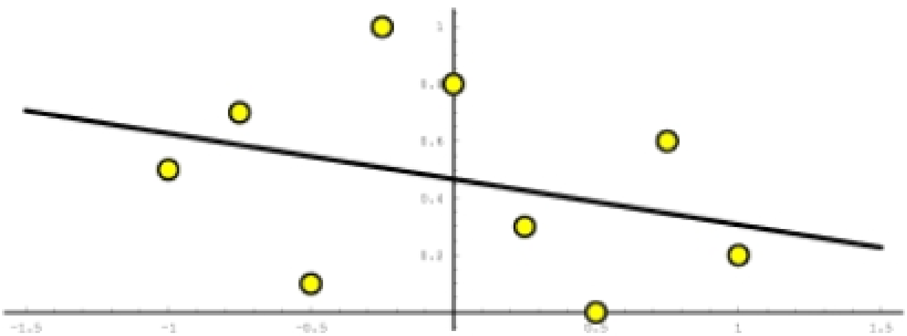
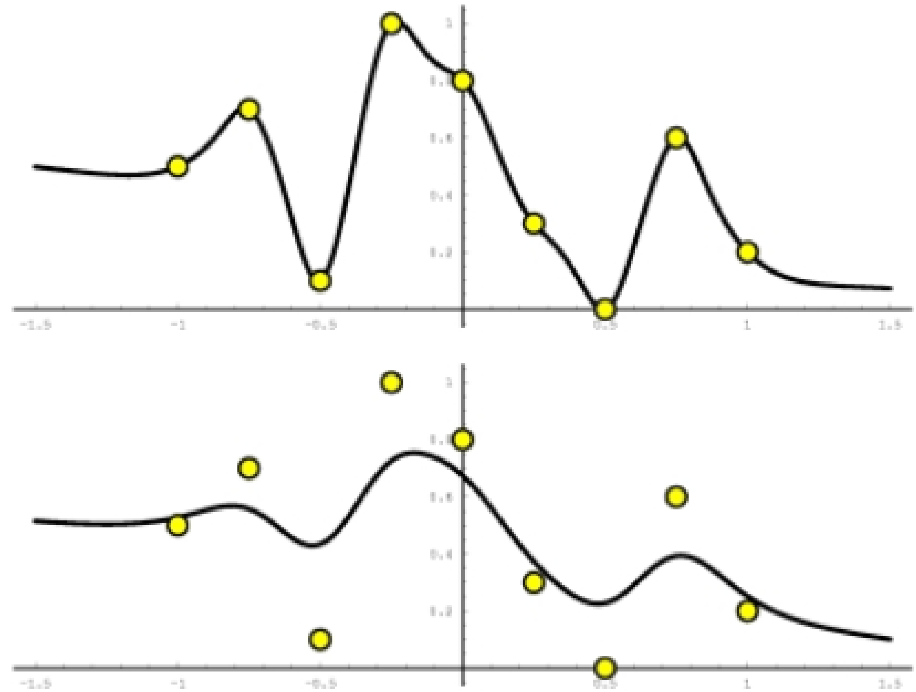
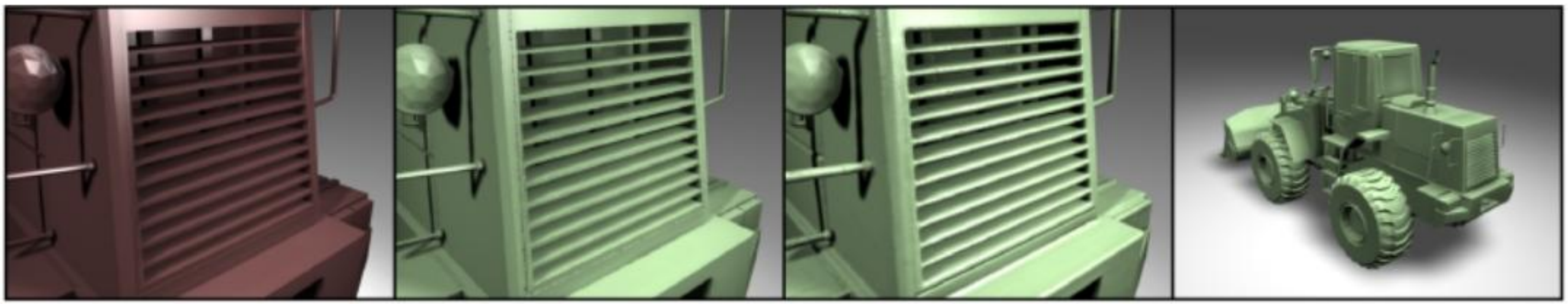
Least Square



Moving Least Square



Moving least square method is an extension of standard least square fit method. It can provide either interpolating or approximating behaviors.



The far-left image shows a closeup view of the original polygons for the heavy-loader's back grill. The center-left image shows the resulting interpolating surface, and the center-right a slightly approximating one. The far-right image shows a rear view of the interpolating surface for the entire loader. The dented appearance near sharp edges is a polygonization artifact.

Standard Least-Squares .

- N points located at positions p_i
  Find a function, f(x), that approximates the values ϕ_i at those points.

$$f(x) = b^{\mathsf{T}}(x)\, c \quad . \qquad (2)$$

$$\begin{bmatrix} b^{\mathsf{T}}(p_1) \\ \vdots \\ b^{\mathsf{T}}(p_N) \end{bmatrix} c = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix} \quad , \qquad (1)$$

where b(x) is the vector of basis functions we use for the fit, and c is the unknown vector of coefficients.

- Find f(x) using standard least-squares:

$$R^2 \equiv \sum [y_i - f(x_i, a_1, a_2, \ldots, a_n)]^2$$

The condition for R^2 to be a minimum is that

$$\frac{\partial (R^2)}{\partial a_i} = 0$$

Moving Least-Squares .

➢ we allow the fit to change depending on where we evaluate the function so that c varies with x.

$$\begin{bmatrix} w(x,p_1) & & \\ & \ddots & \\ & & w(x,p_N) \end{bmatrix} \begin{bmatrix} b^{\mathsf{T}}(p_1) \\ \vdots \\ b^{\mathsf{T}}(p_N) \end{bmatrix} c = \begin{bmatrix} w(x,p_1) & & \\ & \ddots & \\ & & w(x,p_N) \end{bmatrix} \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_N \end{bmatrix} \tag{3}$$

where $w(x, p_i) = w(\|x - p_i\|)$.

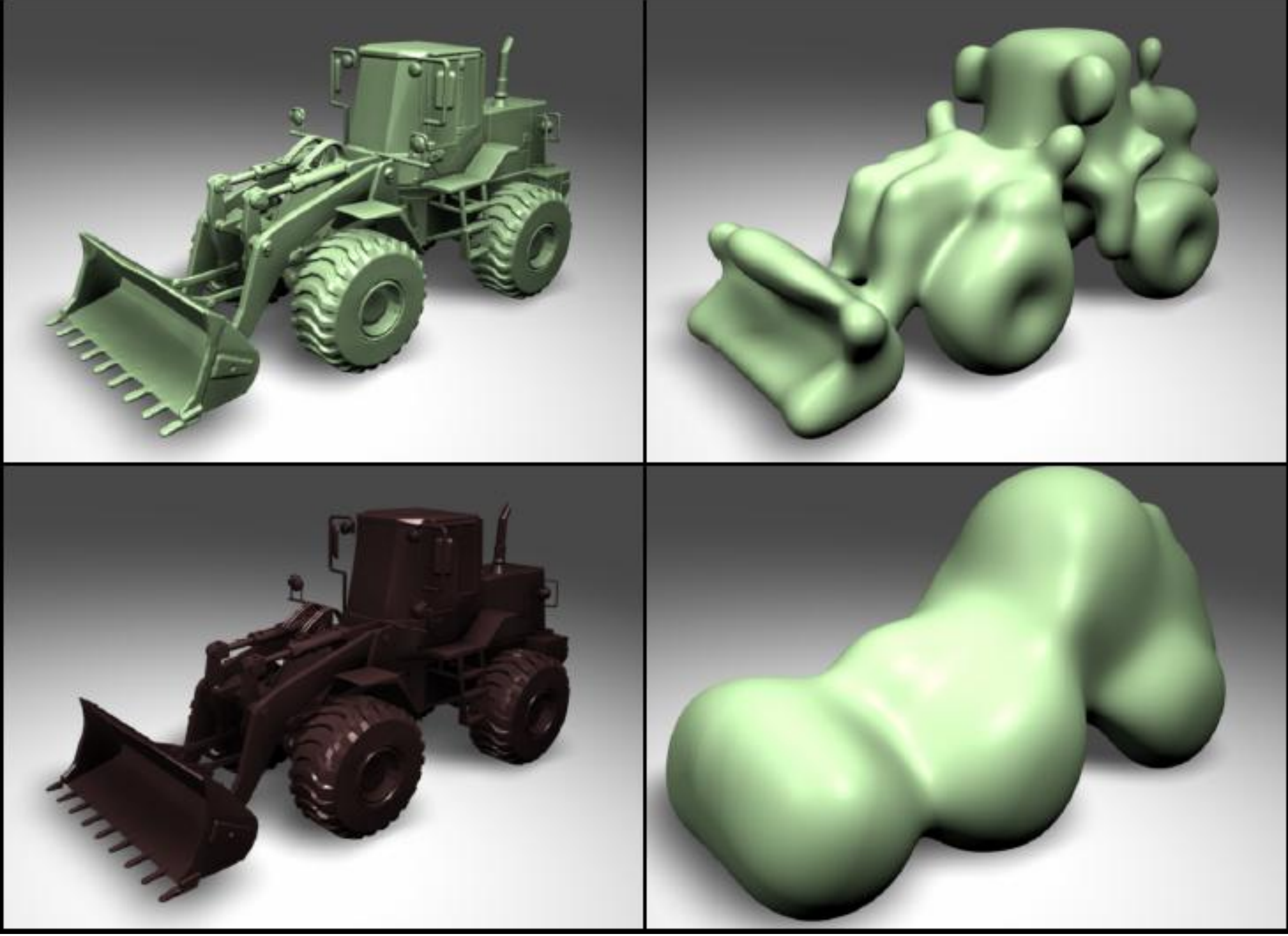
$$w(r) = \frac{1}{(r^2 + \epsilon^2)} \quad . \tag{4}$$

➢ Find f(x) using standard least-squares:

weighted least-square error

$$\sum_{i \in I} (p(x) - f_i)^2 \theta(\|x - x_i\|)$$

over all polynomials p of degree m in $\mathbb{R}^n$. $\theta(s)$ is the weight and it tends to zero as $s \to \infty$.



Reference

- http://en.wikipedia.org/wiki/Moving_least_squares
- http://en.wikipedia.org/wiki/Least_squares
- http://mathworld.wolfram.com/LeastSquaresFitting.html
- SHEN C., O'BRIEN J., SHEWCHUK J.: Interpolating and approximating implicit surfaces from polygon soup. *TOG (SIGGRAPH '04) 23 (2004), 896–904.*